Robust Control Problem as H2 and H∞ control problem applied to the robust controller design of Active Queue Management routers for Internet Protocol

Ichrak Tolaimate, Nourredine Elalami

Abstract— During the last years, the Internet Application (Remote Flexible Control and Instrumentation, Telecontrol...) has retained the attention of Control researchers for modeling and studying the congestion control of TCP/IP (Transfer Control Protocol/Internet Protocol) networks. The controllers proposed in the literature such as RED (Random Early Detection) or PID (Proportional Integrate Derivative) or H2/H∞ (H2 norm/H infinity norm), give good performance under certain conditions, but they become unstable if the input delay or/and the parameters of the networks change beyond some limits. Different papers have treated the H2/H∞ robust control problem, but in this paper, adding to formulate and solve the standard robust control problem as an H2/H∞, and synthesize corresponding controllers H2, H∞ and mixed sensitivity controller, we will show how the Routh –Hurwitz criterion is weak in front of the small gain theorem, and does not guarantee the robustness of the system.

Keywords— AQM, controller, fragility, H2, H∞, Robust Control, Routh-Hurwitz, sensitivity, TCP.

I. Introduction

THE congestion in TCP happens when the demand in resource allocation is greater than the network capacity or similarly when the packet flow in a link is greater than the link capacity. In this case, packet drop or retransmission deteriorates the quality of service offered by the network.

The modern theory of congestion control was pioneered by Frank Kelly, who applied microeconomic theory and convex optimization theory to describe how individuals controlling their own rates can interact to achieve an "optimal" network-wide rate allocation.

Congestion control concerns controlling traffic entry into a telecommunications network, so as to avoid congestive collapse and oversubscription of any processing or link capabilities of the intermediate nodes and networks and taking resource reducing steps, such as reducing the rate of sending packets.

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The congestion-control mechanism, necessary in an under-charged network, becomes indispensable in an over-charged network. Without the congestion-control mechanism, we may not only miss the quality of service but also fail in performing the service which means that the network may enter a blocking state. Different congestion-control mechanisms exist. They differ by their level action (in the network layer) and their algorithms. They may be classified into two main families:

- End to end congestion-control
- Closed loop congestion-control

The first class uses only the information sent by the end extremity to update the packet flow rate.
The second congestion control algorithm update the source flow rate to maintain a predefined constant level of gateway buffer and so offering a better quality of service to the users[10]- [13]- [14]- [12].

In fact, the traffic congestion of the Internet is one of the major communication problems lived by millions of users. Many works have been devoted to improve the internet congestion control performance. Earliest efforts were focused on TCP (Transmission Control Protocol) enhancement. Recently, several mathematical models of AQM (Active Queue Management) schemes supporting TCP flows in communication networks have been proposed, for the purpose of alleviating congestion problem for Internet Protocol network and providing some notion of Quality of Service (QoS). From these models, a control theory based approach can be used to design AQM schemes [2] - [4] - [5].

We follow the model introduced in [3]: Fig. 1 shows the theory system structure of the model for wired network and wired–wireless network, and the wired–wireless architectural trend in enterprise 802.11 deployments is shown in Fig. 2, which is included in Fig. 1.

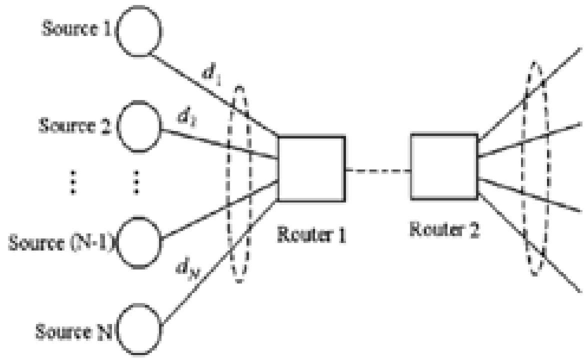

Fig.1 The theory system network model.

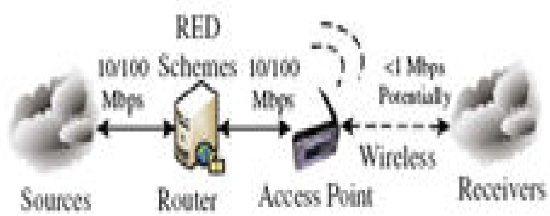

Fig.2 Wired-Wireless architectural trend in entreprise802.11 deployements

As shown in Fig. 1, TCP sources send data packets passing through the routers to their corresponding destinations. The data will be buffered in these routers. The buffer will decide the data packet drop probability p based on the congestion of the current queue. And then it computes p to drive packet dropping. The sending window size of TCP Sender at next time slot will be adjusted based on acknowledgements of Receiver [3].

At follows, we will detail the TCP model.

II. TCP/AQM MODEL

In fact, a fluid model of TCP dynamical behavior was developed; it uses the theory of stochastic differential equations. The model describes the evolution of the variables on the network such as TCP Window size and Queue length. Fig. 3 shows the links between the variables on the network [3] - [8]:

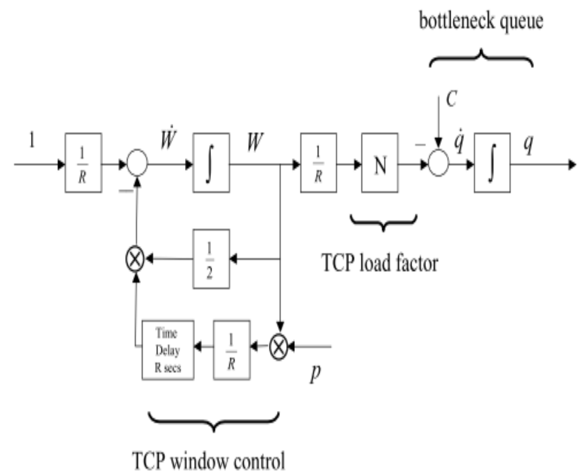

Fig.3 Block-Diagram of TCP connection

Based on some reasonable assumptions, we get the following relations [9]:

$$
\begin{cases}
\dot{W}(t) = \dfrac{1}{\tau(t)} - \dfrac{W(t)}{2}\dfrac{W(t-\tau(t))}{\tau(t-\tau(t))}p(t-\tau(t)), & (1)\\
\dot{q}(t) = \begin{cases} -C(t) + \dfrac{N(t)}{\tau(t)}W(t), \\ when\ : q(t) \succ 0, \\ \max\left\{0, -C(t) + \dfrac{N(t)}{\tau(t)}W(t)\right\}, \\ when\ : q(t) = 0 \end{cases} & (2)\\
\tau(t) = \dfrac{q(t)}{C(t)} + T_p, & (3)
\end{cases}
$$

Where:

TABLE I

Description of Network parameters

Parameter	Description
W(t)	Window length
$\dot{W}$(t)	Time Derivate of Window length
q(t)	Queue length
C	Transmission Capacity
R	RTT made up of two items
Tp	Propagation delay
N	Number of TCP connections
P	Probability of packet dropping

The equilibrium point is defined by:

$\dot{w}(t) = 0$ and $\dot{q}(t) = 0$, and the linearization of (1) and (2) at this point gives:

$$\delta \dot{w}(t) = \frac{-2Nw(t)}{R^2C} - \frac{Rc^2}{2N^2}p(t-R) \tag{4}$$

$$\delta \dot{q}(t) = \frac{Nw(t)}{R} - \frac{q(t)}{R} \tag{5}$$

We apply Laplace Transformation to (4) and (5), and, then we get:

$$sW(s) = \frac{-2NW(s)}{R^2C} - \frac{RC^2}{2N^2}e^{-Rs}\ p(s)$$

$$sQ(s) = \frac{NW(s)}{R} - \frac{Q(s)}{R}$$

This leads to:

$$G(s) = \frac{Q(s)}{p(s)} = \frac{ke^{-Rs}}{(T_1 s+1)(T_2 S+1)} \tag{6}$$

Where: $k = \frac{(RC)^3}{4N^2}; T_1 = \frac{R^2C}{2N}; T_2 = R$

So, the linearization of equations can be modeled as a two-order with delay plant transfer function.

As demonstrated in [9], the second order of Padé approximation is adequate to approach the true system.
We will consider the Second order Padé approximation which is given by:

$$e^{-Rs} = \frac{1 - \frac{Rs}{2} + \frac{(Rs)^2}{12}}{1 + \frac{Rs}{2} + \frac{(Rs)^2}{12}} \tag{7}$$

And apply it to (6);
So, the new Transfer function is:

$$Gs(s) = \frac{k(1 - \frac{Rs}{2} + \frac{(Rs)^2}{12})}{(T1s+1)(T2s+1)(1 + \frac{Rs}{2} + \frac{(Rs)^2}{12})} \tag{8}$$

This plant can be written as:

$$Gs(s) = \frac{b_0 + b_1 s + b_2 s^2 + b_3 s^3}{p_0 + p_1 s + p_2 s^2 + p_3 s^3 + s^4} \tag{9}$$

With:

$$b_0 = \frac{24k}{R^5c^2}; b_1 = \frac{12k}{R^4c}; b_2 = \frac{2k}{R^3c}; b_3 = 0$$

And:

$$p_0 = \frac{24N}{R^5c}; p_1 = \frac{36N}{R^4c} + \frac{12}{R^3};$$

$$p_2 = \frac{14N}{R^3c} + \frac{18}{R^2}; p_3 = \frac{2N}{R^2c} + \frac{7}{R}$$

So then, we will quietly study the robust control problem as H2 and H∞ control problem.

III. ROBUST CONTROL PROBLEM AS H2 AND H∞ CONTROL PROBLEM

A. Introduction

The term H∞ ("H-infinity") comes from the name of the mathematical space over which the optimization takes place: H∞ is the space of matrix-valued functions that are analytic and bounded in the open right-half of the complex plane defined by Re(s) > 0; the H∞ norm is the maximum singular value of the function over that space. (This can be interpreted as a maximum gain in any direction and at any frequency; for SISO systems, this is effectively the maximum magnitude of the frequency response.)

H_∞ methods are used in control theory to synthesize controllers achieving robust performance or stabilization. To use H_∞ methods, a control designer expresses the control problem as a mathematical optimization problem and then finds the controller that solves this. H_∞ techniques have the advantage over classical control techniques in that they are readily applicable to problems involving multivariable systems with cross-coupling between channels; disadvantages of H_∞ techniques include the level of mathematical understanding needed to apply them successfully and the need for a reasonably good model of the system to be controlled. Problem formulation is important, since any controller synthesized will only be 'optimal' in the formulated sense: optimizing the wrong thing often makes things worse rather than better. Also, non-linear constraints such as saturation are generally not well-handled.

As well, H∞ techniques can be used to minimize the closed loop impact of a perturbation: depending on the problem formulation, the impact will either be measured in terms of stabilization or performance.
Simultaneously, optimizing robust performance and robust stabilization is difficult. One method that comes close to achieving this is H_∞ loop-shaping, which allows the control designer to apply classical loop-shaping concepts to the multivariable frequency response to get good robust performance, and then optimizes the response near the system bandwidth to achieve good robust stabilization.

[1]-[11] In fact, the H∞ theory provides a direct synthesizing controller which optimally satisfies singular value loop shaping specifications. The standard setup of the control problem consist of finding a static or dynamic feedback controller such that the H∞ norm (a uncertainty) of the closed loop transfer function is less than a given positive number under constraint that the closed loop system is internally stable.

The advantages of the proposed linear robust controller are address stability and sensitivity, exact loop shaping, direct one-step procedure and closed-loop always stable [11].

B. Problem formulation

[1] To formulate and solve a robust control problem as an H2 or H∞ control problem, let us first present the small-gain theorem.

Consider a system with uncertainty. Let us assume that we can separate the uncertainty from the nominal system in a feedback loop, as shown in Figure 4.

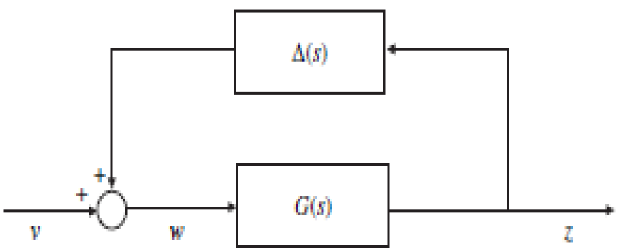

Fig.4 uncertainty and small gain theory

In Figure 4, G(s) is the transfer function of the nominal system; and Δ(s) is the uncertainty. v and z are the input and output of the overall perturbed system. w is the input of the nominal system.

Reconsider our system with the following general transfer function:

$$Gs(s) = \frac{b_0 + b_1 s + b_2 s^2 + b_3 s^3}{p_0 + p_1 s + p_2 s^2 + p_3 s^3 + s^4} \tag{9}$$

Where the uncertainty is described by pi ϵ[pi$^-$ pi+] and i = 0,1,2,3,4.
We can find its controllable canonical realization as:

$$\begin{cases} \dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -p_0 & -p_1 & -p_2 & -p_3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U \\ Y = \begin{bmatrix} b_0 & b_1 & b_2\, b_3 \end{bmatrix} X \end{cases} \tag{10}$$

Denote $p_i^0 = \frac{p_i^- + p_i^+}{2}$ and $p_i = p_i^0 + \Delta p_i$

With $\Delta p_i \in [-\frac{p_i^- - p_i^+}{2}; \frac{p_i^- - p_i^+}{2}]$

Then we can re-write the state equation as:

$$\begin{cases} \dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -p_0^0 - \Delta p_0 & -p_1^0 - \Delta p_1 & -p_2^0 - \Delta p_2 & -p_3^0 - \Delta p_3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U \\ Y = \begin{bmatrix} b_0 & b_1 & b_2\, b_3 \end{bmatrix} X \end{cases}$$

$$\begin{cases} \dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -p_0^0 & -p_1^0 & -p_2^0 & -p_3^0 \end{bmatrix} X + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\Delta p_0 & -\Delta p_1 & -\Delta p_2 & -\Delta p_3 \end{bmatrix} X \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U \\ Y = \begin{bmatrix} b_0 & b_1 & b_2\, b_3 \end{bmatrix} X \end{cases} \tag{11}$$

Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -p_0^0 & -p_1^0 & -p_2^0 & -p_3^0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} -\Delta p_0 & -\Delta p_1 & -\Delta p_2 & -\Delta p_3 \end{bmatrix}$$

$$C = \begin{bmatrix} b_0 & b_1 & b_2\, b_3 \end{bmatrix}$$

Define e = Δx.
Then we have the following equations:

$$\dot{X} = AX + B\Delta X + Bu = AX + B(u+e) \tag{12}$$
$$Y = CX$$

In other words, we can translate the system into the one in Figure 5.

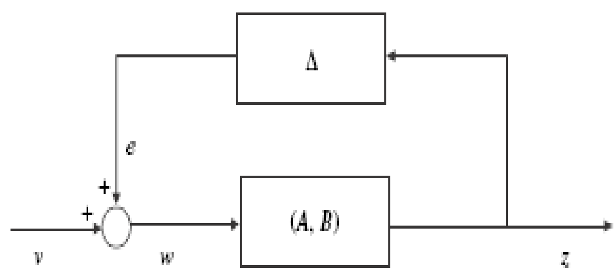

Fig.5 Separation of the uncertainty and the system

The Fig.6 presents the model of TCP/IP flow under multiplicative form separated from uncertainty. It is equivalent to the model presented in Fig 5.

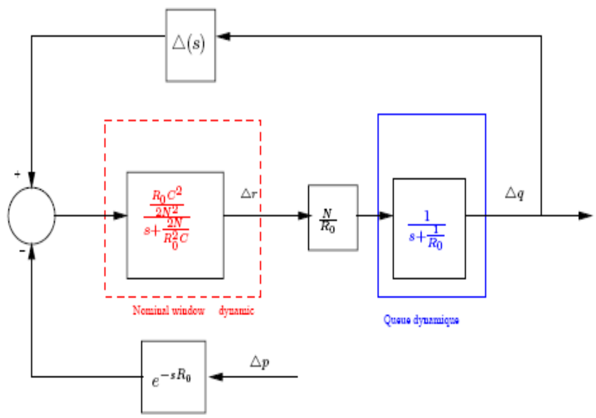

Fig.6 Block diagram representation of model uncertainties of TCP/IP flow under uncertainties

The problem we want to investigate is as follows: Assume that the nominal system G(s) is stable and how big can the uncertainty be, before the perturbed system becomes unstable? In other words, what is the bound on the uncertainty Δ(s) that guarantees the stability of the perturbed system?
This question is partially answered by the following small-gain theorem.

C. .The Theorem of small Gain

Consider the system in Figure 4:
Let G(s) be a proper real rational stable transfer function. Assume that $\|G\|\infty \prec \lambda$ for some λ>0. Then the perturbed (closed-loop) system is stable for all proper real rational stable transfer functions Δ(s) such that Δ(s) $\leq 1/\lambda$.

D. Application to TCP/AQM Model

Let's

$R \in [0.06; 0.9]$ s,

$C \in [15; 60]$ Mb/s,

$N \in [10; 100]$

So,

$p_0 \in [6.7740\times10^{-5}; 205.7613]$;

$p_1 \in [16.4610; 5.5574\times10^{4}]$;

$p_2 \in [22.2223; 5000]$;

$p_3 \in [7.7778; 116.6704]$

And,

$p_0^0 = 102.8807$;
$p^1_0 = 2.7795\times10^4$;
$p^2_0 = 2.5111\times10^3$;
$p^3_0 = 62.2241$;

And,

$\Delta p_0 \in [-102.8806; 102.8806]$;

$\Delta p_1 \in [-2.7779\times10^4; 2.7779\times10^4]$;

$\Delta p_2 \in [-2.4889\times10^3; 2.4889\times10^3]$;

$\Delta p_3 \in [-54.4463; 54.4463]$

Replacing with proposed values in (11), we get:

$$\begin{cases} \dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1028806-\Delta p_0 & -2.7779x10^4-\Delta p_1 & -2.4889x10^3-\Delta p_2 & -54.4463-\Delta p_3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U \\ Y = \begin{bmatrix} b_0 & b_1 & b_2 \, b_3 \end{bmatrix} X \end{cases}$$

And the nominal system is given by:

$$\begin{cases} \dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -102.8806_0 & -2.7779x10^4_1 & -2.4889x10^3 & -54.4463 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U \\ Y = [b_0 \quad b_1 \quad b_2\, b_3] X \end{cases}$$

The poles of this system are:
-23.3249 +35.2202i; -23.3249 -35.2202i; -15.5706 and -0.0037

So, the nominal system is stable. Let G(s) be the transfer function of the nominal system;
We have:

$$G(s) = \frac{1.125x10^{12}\, s^2 + 1.125x10^{14}\, s + 4.5\, x10^5}{s^4 + 62.22\, s^3 + 2511\, s^2 + 2.78\, x10^4\, s + 102.9}$$

Then we can calculate the $\|H\|\infty$ norm of G(s):
Using Scilab, we obtain:
$\|G\|\infty < 4.0479x10^9$

Since the $\|H\|\infty$ norm of $\Delta = [-\Delta p_0 \quad -\Delta p_1 \quad -\Delta p_2 \quad -\Delta p_3]$

$is \sqrt{\Delta p_0^2 + \Delta p_1^2 + \Delta p_2^2 + \Delta p_3^2}$

And by the theorem of small gain already announced, the perturbed system is stable for all uncertainties such that:

$$\|\Delta\|\infty = \sqrt{\Delta p_0^2 + \Delta p_1^2 + \Delta p_2^2 + \Delta p_3^2}$$

$$\leq 1/4.0479x10^9 = 2.4704x10^{-10}$$

Or

$$\Delta p_0^2 + \Delta p_1^2 + \Delta p_2^2 + \Delta p_3^2 \leq 6.1029x10^{-20} \qquad (13)$$

Make a note of that this condition is actually very conformist. To see this, let us write the characteristic equation of the perturbed system as:

$$s^4 + (62.22+\Delta p_3)s^3 + (2511 + \Delta p_2)s^2 + (2.78\, x10^4 + \Delta p_1)s + (102.9+\Delta p_0) = 0 \qquad (14)$$

Let us apply the Routh–Hurwitz criterion to this characteristic equation:
We know that the perturbed system is stable if and only if:

1>0; 62.22+ Δp_3 >0; 102.9+ Δp_0 >0; (2511 + Δp_2)(62.22+ Δp_3) > (2.78 x10^4 + Δp_1);
And
(2.78 x10^4 + Δp_1 (2511+ Δp_2)>(102.9+ Δp_0) (62.22+ Δp_3)
For example, if we take:
$\Delta p_0 = 20$
$\Delta p_1 = 100$
$\Delta p_2 = 50$
$\Delta p_3 = 10$
The conditions cited up are verified, and we have for this example:

$$\Delta p_0^2 + \Delta p_1^2 + \Delta p_2^2 + \Delta p_3^2 = 13000 \qquad (15)$$

This is much greater than 6.1029x10^{-20}
This condition (15) is much weaker than Condition (13).
To this point, we have discussed analysis problems; that is, given a perturbed system with bounds on the uncertainty, we can check if the condition in the theorem of small gain is satisfied. If the condition is satisfied, then the robust stability is guaranteed; if not, the system may or may not be robustly stable.
In the next section, we will turn to synthesis problem; that is to design a controller that will achieve robust stability for the largest bounds on the uncertainty.

IV. H2/H∞ CONTROL SYNTHESIS

A. Introduction

[1] Before we discuss the H2/H∞ control synthesis, let us first mention that the H2/H∞ approach is very different from the optimal approach: In the optimal control approach, we start with the bounds of uncertainties. We then design a controller based on these bounds. As the result, if the controller exists, then it is guaranteed to robustly stabilize the perturbed system. On the other hand, in the H2/H∞ approach, the bounds on uncertainties are not given in advance. The synthesis will try to achieve the largest tolerance range on uncertainty. However, there is no guarantee that the range is large enough to cover all possible uncertainties.
Initially, the H∞/H2 approach is based in transfer function model. Results are obtained using transfer functions in the frequency domain. Late, it was found the H∞/H2 approach can be effectively presented using the state space model of systems. The state space model is what we use in this paper because it is simpler to use the state space model to handle with multivariable systems with multiple inputs and multiple outputs.

B. Formulation

To formulate the H2/H∞ approach, let us consider the setting in Figure 4, but assume the G(s) can now be modified by introducing a controller as shown in Figure 6.
In Figure 7, F(s) is the transfer function of the plant; K(s) is the transfer function of the controller to be designed; u is the input for control; and y is the output (measurement) for control.

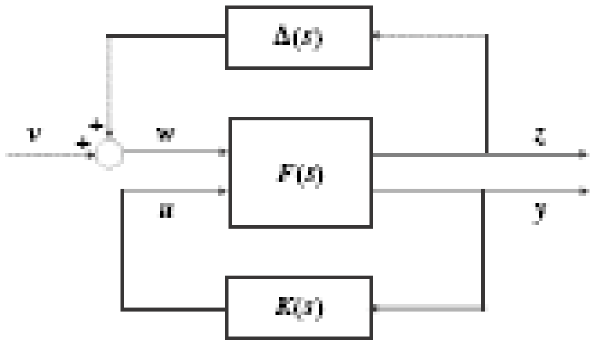

Fig 7. H2/H∞ approach: introduction of a controller to minimize the H2/H∞ norm

Comparing Figure 4 with Figure 7, we see that nominal system G(s) is now equivalent to the controlled system consisting of the plant F(s) and the controller K(s). The plant F(s) is given while the controller K (s) is to be designed.

From the previous discussions, we know that in order to maximize the tolerance range on uncertainty, we need to design a practical controller that minimizes the norm of the transfer function from w to z.

The H∞ synthesis is carried out in two phases:

The H∞ formulation procedure:

The robustness to modeling errors and weighting the appropriate input – output transfer functions reflects usually the performance requirements. The weights and the dynamic model of the power system are then augmented into an H∞ standard plant.

The H∞ solution:

The second phase is the H∞ solution. In this phase the standard plant is programmed by computer design software such as Matlab or Scilab that we will use in this case.

Time response simulations are used to validate the results obtained and illustrate the dynamic system response to state disturbances [11].

C. Results of H2/H∞ control synthesis Applied to the TCP/AQM Model:

- H2 controller:

Using Scilab, we can synthesize an H2 controller for the TCP /AQM model, under assumptions mentioned before.

The H2 controller representation is as:

$$\dot{\phi}=\begin{bmatrix} -1.\_824 & 0.2037 & 0.3727 & -0.3266 \\ 0.1665 & -1.483 & 0.5482 & -0.4707 \\ 0.1701 & -0.2321 & -2.147 & 0.3919 \\ 0.1098 & -0.9916 & -0.056 & -1.151 \end{bmatrix}\phi+\begin{bmatrix} 0.1059 \\ 0.089 \\ -0.001 \\ -0.3282 \end{bmatrix}Y$$

$$\beta=\begin{bmatrix} 0.078 & 0.3795 & 0.3128 & -0.036 \end{bmatrix}\phi$$

And it has as a transfer function:

$0.05398\ s^3 + 0.2977\ s^2 + 0.5004\ s + 0.2353$

$s^4 + 6.605\ s^3 + 15.63\ s^2 + 15.55\ s + 5.462$

We can verify that the closed loop is stable, in fact we have
pole(H2Closed Loop)= -0.4045; -0.7841; -0.7899 ; -1.4775
-2.0386; -2.1866 + 0.0891i;
-2.1866 - 0.0891i; -2.3379

So then, we can simulate the time response of the system, and verify the system obtained with H2 controller is stable.

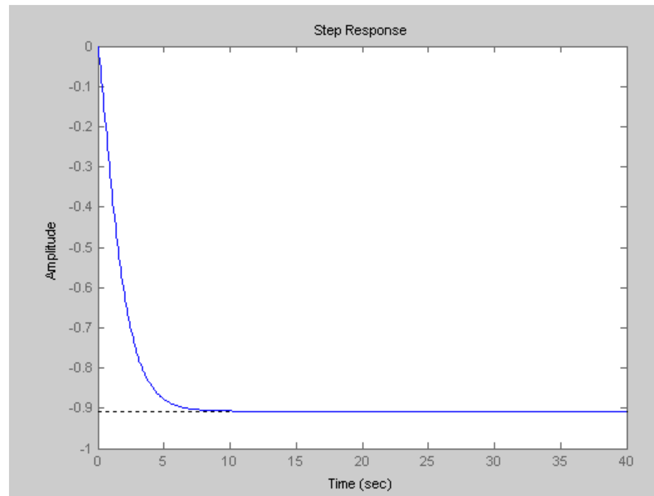

Fig 8. Step response of the closed loop obtained wit H2 controller

- H∞ controller

The H∞ control problem is similar, but more complicated if we use standard calculs. We can synthesize an 'optimal' H∞ controller that internally stabilizes the controlled system and 'minimizes' the H∞ norm G(s) of the transfer function G(s) from w to z.

In our case, we will use Scilab with a tolerance of 1%, and we obtain the following H∞ controller:
This is done by using an algorithm of bisection similar to the one used in calculating the H∞ norm.

Using Scilab, the H∞ controller representation is as:

$$\dot{\phi}=\begin{bmatrix} -11.17 & -9.305 & -11.59 & 1.738 \\ 33.61 & 32.54 & 43.36 & -5.6 \\ -36.81 & -37.76 & -49.37 & 6.654 \\ -73.12 & -75.44 & -93.74 & 10.45 \end{bmatrix}\phi+\begin{bmatrix} -0.062 \\ 0.8628 \\ 0 \\ 1.604 \end{bmatrix}Y$$

$$\beta=\begin{bmatrix} 10.88 & 11.23 & 14.03 & -1.857 \end{bmatrix}\phi$$

And it has as a transfer function:

$11.99\ s^3 + 61.86\ s^2 + 98.28\ s + 44.7$

$s^4 + 17.55\ s^3 + 140.3\ s^2 + 409.9\ s + 385$

We can verify that the closed loop is stable, in fact we have:
Pole (H∞ Closed Loop)= -22.4799; -3.3604 ; -0.4045 ;
- 0.7841; -2.038; -2.1866 + 0.0891i;
-2.1866 - 0.0891i; -2.3379

So then, we can simulate the time response of the system, and verify the system obtained with H∞ controller is stable.

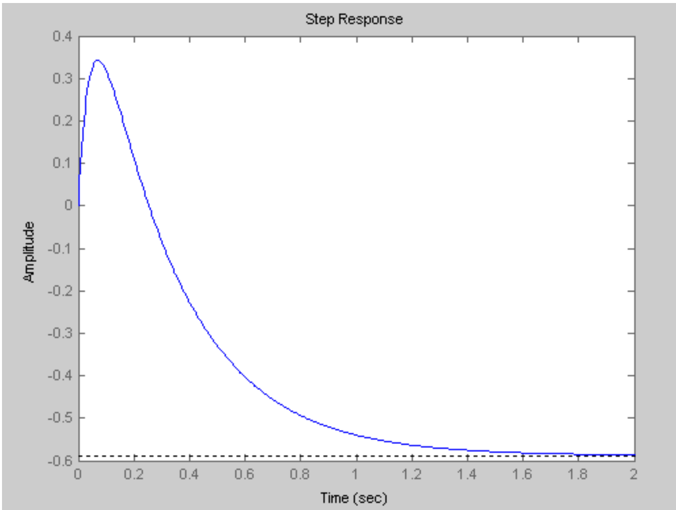

Fig 9. Step response of the closed loop obtained wit H∞ controller

- H_∞ mixed-sensitivity synthesis

The H_∞ mixed-sensitivity synthesis is a method for robust control loop shaping design in modern control theory. It combines the traditional intuition of classical control methods, such as Bode's sensitivity integral, with H-infinity optimization techniques to achieve controllers whose stability and performance properties hold good in spite of bounded differences between the nominal plant assumed in design and the true plant encountered in practice. Essentially, the control system designer describes the desired responsiveness and noise-suppression properties by weighting the plant transfer function in the frequency domain; the resulting 'loop-shape' is then 'robustified' through optimization. Robustification usually has little effect at high and low frequencies, but the response around unity-gain crossover is adjusted to maximize the system's stability margins. H-infinity loop-shaping can be applied to multiple-input multiple-output (MIMO) systems

[1]- [15] The design of robust loop shaping H∞ controllers based on a polynomial system philosophy has been introduced by Kwakernaak and Grimbel. It has been successfully deployed in industry. In 1995, R. Hyde, K. Glover and G. T. Shanks published a paper describing the successful application of the technique to a VSTOL aircraft. In 2008, D. J. Auger, S. Crawshaw and S. L. Hall published another paper describing a successful application to a steerable marine radar tracker, noting that the technique had the following benefits:

- Easy to apply – commercial software handles the hard math.

- Easy to implement – standard transfer functions and state-space methods can be used.
- Plug and play – no need for re-tuning on an installation-by-installation basis.

The controller computed minimizes the H_∞ norm of the closed-loop transfer function mixed with the weighted sensitivity $W_1(s)$, $W_2(s)$, and $W_3(s)$ that penalize the error signal, control signal and output signal respectively. So that, the closed-loop transfer function matrix is the weighted mixed sensitivity as presented in fig 10.

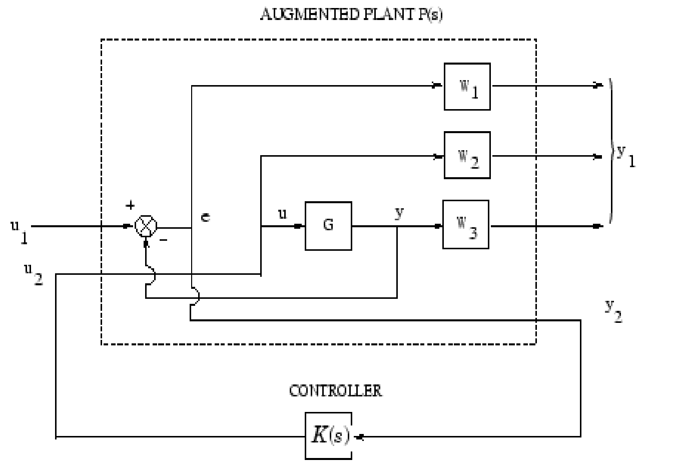

Fig 10. Closed loop transfer function for mixed sensitivity

We call S and T the sensitivity and complementary sensitivity; S, R and T are given by:

S= $(I+ GK)^{-1}$
R= $K(I+GK)^{-1}$
T= $GK(I+GK)^{-1}$

We apply the H_∞ mixed-sensitivity synthesis method to the TCP/AQM model, and then we get:
The controller has as a presentation:

$$\dot{\phi}=\begin{bmatrix} -0.01 & 0 & 0 & 0 & 0 \\ 3.808*10^6 & -3.198*10^5 & -7.98*10^8 & 4.99*10^9 & -79.82 \\ 0 & 64 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0 \end{bmatrix}\phi+$$

$$\begin{bmatrix} 0.1346 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} Y$$

$$\beta=\begin{bmatrix} 33.73 & -2.832 & -7077 & -4.42*10^4 & 0 \end{bmatrix}\phi$$

And in fig 11, we have the time response of this system; we verify that the system obtained with mixed H∞ controller is stable.

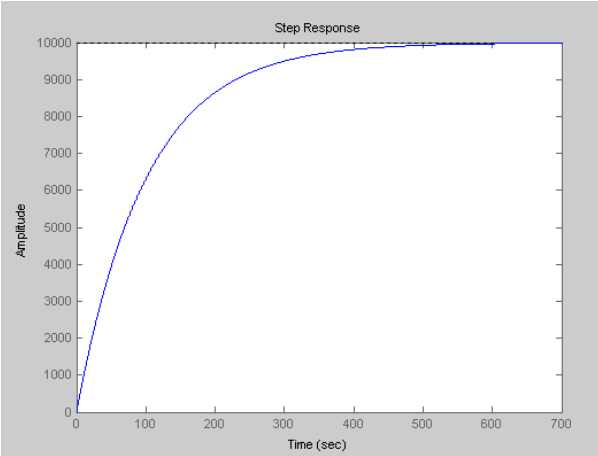

Fig 11. Step response of the closed loop obtained wit mixed H∞ controller

V. Conclusion

This paper presents different computations and simulations of controllers applied to the TCP/AQM networks based on fluid flow models with delay. These controllers correspond to the norms: the H2 norm, the H infinity norm, and the mixed sensitivity norm .We performed a stability analysis of some classes of TCP/AQM networks based on fluid flow models with delay. In a first time, we formulate and solve a robust control problem as an H2 or H∞ control problem, we choose a set of uncertainties for the internal values R, C and N, and apply the elegant theorem of the Small Gain ,so, we guarantee the robust stability of the system for some bound of uncertainty . To detail more, we demonstrate that the condition of small gain theorem is far stronger than the condition of the Routh Hurwitz criterion. To end with, we turn to synthesis problem, and we design different H controllers that achieve robust stability for the largest bounds on the uncertainty.

However, in a next study, we can show by examples that optimum and robust controllers, designed by using the H2, H∞ formulations, can produce extremely fragile controllers, in the sense that small perturbations of the coefficients of the designed controller can destabilize the closed-loop control system. So, we conclude that it is important to keep the nonoptimal design techniques and make an effort to enhance them.

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