# Optimization of Buffer-size Allocation Using Dynamic Programming

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*Abstract*— Assembly-like Queueing systems are used in the assembly processes in production lines in the chemical industry as well as dataflow in computer systems. While many models are constructed in tandem and merge systems, assembly-like systems are known to be more complicated and difficult to analyze. These systems are not investigated in queueing theory. Whereas most research focuses on simple assembly-nodes, in this paper, we evaluate by dynamic programming using numerical analysis to propose buffer-size optimal allocation algorithm.

*Keywords*— Assembly-like system, Buffer Allocation, Dynamic Programming, Production System

## 1. INTRODUCTION

ASSEMBLY processes arise in many practical situations , including assembly-line in productions. Compared to queueing systems with a tandem and merge configurations, assembly-like queueing systems are not simple to analyze. Most research has focused on simple assembly-nodes. Research has attempted to solve an optimal algorithm in assembly-like queueing systems.

In his analysis of network models such as stationary probabilities, Harrison (1973) used infinity models to propose an approximation method in throughput systems.

Mitra (1990) and Tayur (1992) derived the heuristic rule in optimal buffer-size allocation using a tandem system. According to heuristic policy in optimal allocation, an approximation method is considered more effective.

Considering production system in many factories, we should utilize limitation space for buffer-size allocation in each station to distribute it effectively.

At the result of numerical simulation, Hemachandra and Edeupuganti (2003) used the fork-join model to discuss buffer-size allocation to minimize mean waiting time of a typical job or Work-In-Process of system. Different from previous studies on heuristic policies, in this paper we discuss multi-level assembly-like queueing systems to propose a more systematic algorithm to obtain optimal buffer-size allocations using observation and conjecture.
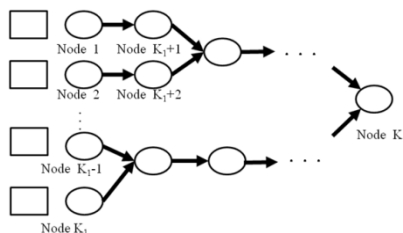
## 2. Assembly-like Queueing Systems

We consider a K-node queueing system as shown in Fig 1. Each node has a single server which provides exponential service with rate $\mu = 4$.

For node 1 through $K_1$, there is an infinite-sized pool of customers in front of each server. Hence, these nodes cannot be idle. Upon service completion at a node, a customer enters a downstream node and receives his service there.

Customers depart from the system after service completion at the last node, Node $K$.

Suppose the input of Node $k$ ($k > K_1$) is the output of Node $u_1(k)$ and $u_2(k)$. Buffers $k_{(1)}$ and $k_{(2)}$ are prepared in Node $k$ for input from each node individually (Fig 2).
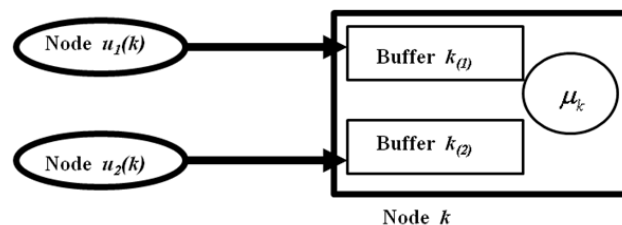


Fig 2. An Assembly-Node

In front of the server, a position is also prepared for each input node individually. The server may begin assembly service for customers in the positions only in the case that there is a customer in each position. Node $k$ cannot begin a service if there is any vacant position in front of the server.
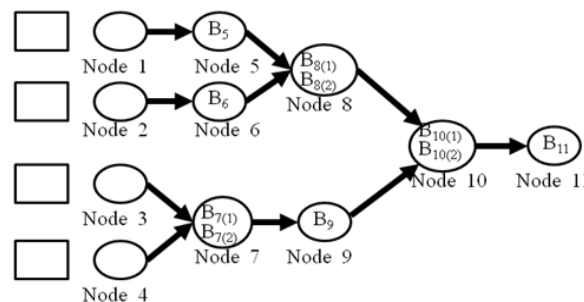


Fig1. Assembly-like Queueing System



Fig3. An 11-Node Model

We define Buffer $k_{(i)}$-1 as the size of Buffer $k_{(i)}$. The server at Node $u_1(k)$ checks state of Buffer $k_{(i)}$ before starting at a new customer service. If Buffer $k_{(i)}$ is full, the customer occupies the server and blocks it.

Service is begun only after a vacant space appears in Buffer $k_{(i)}$. This is called *communicating blocking*. In Fig 3, Buffer depending on Group 1 starts at Buffer 1 lower side set Buffer 2, Buffer 3 and Buffer 4.

Moreover, Group 2 allocates Buffer 5, Buffer 6, and Buffer $7_{(1)}$, Buffer $7_{(2)}$, Buffer $8_{(1)}$, Buffer $8_{(2)}$. Buffer 7 is assembly node. These nodes are shown by the number in bracket.
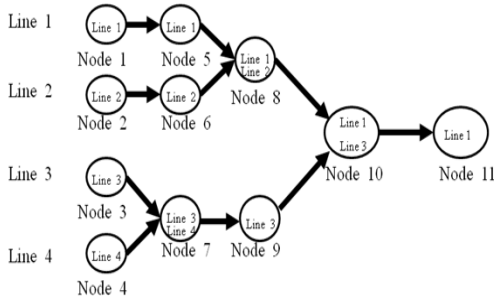


Fig 4. An example of 4 lines

## 3. Observation and Conjecture

This section illustrates a method to approach an optimal throughput in simulation analysis. First, four lines in this model are defined. Second, we simulate this model as buffer allocation in each line. And finally, this system will achieve the optimal buffer-size allocation.

### 3.1Line

We divide the model into 4 lines. Line 1 starts at Node 1 and end at Node $K$. The server in Node 1 consists of Buffer 5, Buffer $8_{(1)}$, Buffer $10_{(1)}$ and Buffer 11.

Line 2 starts at Node 2 and ends at Buffer $8_{(2)}$. Line 3 and Line 4 start from Node 3 and 4 respectively, and end at Buffer $10_{(2)}$ and Buffer $7_{(2)}$ respectively.

An each line does not duplicate (i.e. there is the model existing three assembly-nodes and we need to classify it: Independency of line).

For example, Buffer 8 locating by Node 8 in this system stands for Buffer $8_{(1)}$ and Buffer $8_{(2)}$.

The system in this model defines that the number of buffer and buffer allocation cannot make the adjustment, but buffer-size can make it.

And we consider that achieve the max throughput how to allocate in each buffer, if it is given total buffer-size in this system.

We derive optimal buffer in the following steps.

Step1) Simulated by using assembly-like queuing system
Step2) Searching for regularity of buffer
Step3) Select the best throughput value in getting regularity of buffer

### 3.2Independency of Buffer-size Allocations

For general assembly-like queueing systems, we define the following notations for buffer-size and allocations.

Notations:
$N$: Total buffer size in the system
$L_j$: Line $j$ $(j = 1,2,3,\cdots,K_1)$
$n_j$: Total buffer size of $L_j$ $(j = 1,2,3,\cdots,K_1)$
$\boldsymbol{B}_j(n_j)$: Buffer-size allocation vector of $L_j$ given that the total buffer-size of $L_j$ is $n_j$
$\boldsymbol{B}'_j(n_j)$: Buffer-size allocation vector other than $\boldsymbol{B}_j(n_j)$
$\overline{\boldsymbol{B}}_j(n_j)$: Buffer-size allocation vector of the system other than $L_j$ given $n_j (j = 1,2,3,\cdots,K_1)$
$T\left(\boldsymbol{B}_j(n_j), \overline{\boldsymbol{B}}_j(n_j)\right)$: Throughput of the system given $\boldsymbol{B}_j(n_j)$ and $\overline{\boldsymbol{B}}_j(n_j)$ $(j = 1,2,3,\cdots,K_1)$

Based on the result of simulation, we get the following observation.

**Observation:** Given the total buffer size in $L_j$ is $n_j$, if

$$^\forall \overline{B}_j(n_j): T\left(\boldsymbol{B}_j(n_j), \overline{\boldsymbol{B}}_j(n_j)\right) > T\left(\boldsymbol{B}'_j(n_j), \boldsymbol{B}_j(n_j)\right),$$

then

$$^\forall B'_j(n_j): T\left(\boldsymbol{B}_j(n_j), \overline{\boldsymbol{B}}'_j(n_j)\right) > T\left(\boldsymbol{B}'_j(n_j), \overline{\boldsymbol{B}}'_j(n_j)\right).$$

As an example, in the 11-node model, let $N$=13 and $n_1 = 5$. Table 1 enumerates 4 cases of $B_1(5)$, and Table 2 lists some case for $\overline{B_1}(5)$.

In Fig5, we can see that the line for Case① never cross with Case② or other cases. This is what the above observation implies. And the observation can be regarded as a kind of independency among the lines of the system.

To derive the maximum $\left(\boldsymbol{B}_j(n_j), \overline{\boldsymbol{B}}_j(n_j)\right)$, buffer allocation depend on (1, 1,2,1) in $L_1$.

**Remark.** Applying to observation, suppose we consider buffer allocation in $L_1 \sim L_4$ when total buffer size is 13. Now, the buffer allocation feasible in $L_1$ enumerate with total buffer size. In this case, $L_1$ allocate range of between $\boldsymbol{B}_1(5)$ and $\boldsymbol{B}_1(8)$. Based on simulation result, the highest throughput in $L_1$ is $\boldsymbol{B}_1(5) = (1,1,2,1)$. Allocation vector in $\boldsymbol{B}_1(5)$ note $(1,1,1,2), (1,1,2,1), (1,2,1,1)$ and $(2,1,1,1)$. Next, buffer allocation with $L_2 \sim L_4$ besides $L_1$ buffer allocation in $L_1$ subtract from total buffer size. In figure 3, it noted buffer allocation type that four cases remain the buffer size in $L_2 \sim L_4$.
Compared to $T\left(\boldsymbol{B}_j(n_j), \overline{\boldsymbol{B}}_j(n_j)\right)$ with $T\left(\boldsymbol{B}'_j(n_j) | \overline{\boldsymbol{B}}_j(n_j)\right)$ in $\boldsymbol{B}_1(5)$. In $\boldsymbol{B}_1(5)$, suppose $\boldsymbol{B}_1(5) = (1,1,2,1)$, $\boldsymbol{B}'_1(5) = (2,1,1,1)$, $\boldsymbol{B}_1(5) = (1,1,2,1)$ is the highest throughput in Case $\text{II}\left(\text{i. e. T}\left(\boldsymbol{B}_j(n_j) | \overline{\boldsymbol{B}}_j(n_j)\right) > T\left(\boldsymbol{B}'_j(n_j) | \overline{\boldsymbol{B}}_j(n_j)\right)\right)$ independent of Case I , III, IV$\left(\text{i. e.} T\left(\boldsymbol{B}_j(n_j) | \boldsymbol{B}'_j(n_j)\right) > T\left(\boldsymbol{B}'_j(n_j) | \overline{\boldsymbol{B}}_j(n_j)\right)\right)$. Hence, $\boldsymbol{B}_1(5) = (1,1,2,1)$ is the highest throughput in $(1,1,1,2), (1,1,2,1), (1,2,1,1), (2,1,1,1)$.

Moreover, in $\overline{\boldsymbol{B}}_j(n_j)$, to maximize $T\left(\boldsymbol{B}_j(n_j)|\overline{\boldsymbol{B}}_j(n_j)\right)$, it is essential to buffer allocation (1,1,2,1) in $L_1$ with $\boldsymbol{B}_1(5)$ (Fig.5).

Finding of our study based on Observation derived the highest value of $T\left(\mathbf{B_j}(n_j)|\overline{\mathbf{B}}_j(n_j)\right)$, thus, subject to hold for buffer allocation until $L_j$ and considered maximizing $T\left(\mathbf{B_j}(n_j)|\overline{\mathbf{B}}_j(n_j)\right)$ the addional allocation in $L_j$ using Dynamic Programming.

We search for buffer size feasible in $L_1$ and derive allocation feasible remain $L_j$ with total buffer size.

It means that location of buffer can be existing in each $L_j$ and this algorithm applies to derive buffer size in feasible allocation with $L_j$ and remain $L_j$.

Table1. Buffer-size Allocation in $L_1$

| Case | $B_1(5)$ |
|---|---|
| a | (1,1,1,2) |
| b | (1,1,2,1) |
| c | (1,2,1,1) |
| d | (2,1,1,1) |

Table 2.Buffer-size Allocation in $L_2 \sim L_4$

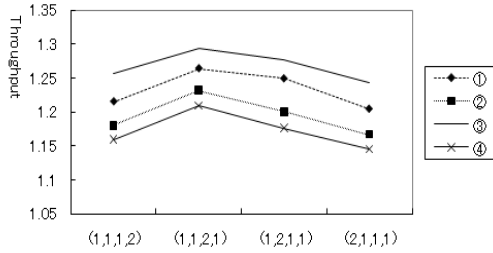| | $B_6$ | $B_{8(2)}$ | $B_{7(1)}$ | $B_9$ | $B_{10(2)}$ | $B_{7(2)}$ |
|---|---|---|---|---|---|---|
| ① | 1 | 1 | 1 | 1 | 2 | 2 |
| ② | 1 | 1 | 2 | 1 | 1 | 2 |
| ③ | 1 | 2 | 1 | 1 | 2 | 1 |
| ④ | 1 | 1 | 1 | 1 | 1 | 3 |



Fig 5.Value of $T\left(B_1(5), \overline{B}_1(5)\right)$

### 3.3 Optimal Buffer Allocation

Using the result of the above observation, we derive the optimal buffer allocation until $L_1$.

Moreover, total buffer-size until $L_j$ denote $s_j$, Buffer-size allocation vector until $L_j$ denote $\overline{\boldsymbol{\beta}}_j(s_j)$. To derive optimal buffer allocation, we define the below.

Notations

$\boldsymbol{\beta}_j(s_j)$: Buffer-size vector from $L_1$ to $L_j$ given $s_j$
$$(j = 1,2,3, \cdots, K_1)$$

$\overline{\boldsymbol{\beta}}_j(s_j)$: Buffer-size vector behind $L_{j+1}$ given $s_j$
$$(j = 1,2,3, \cdots, K_1 - 1)$$

$\boldsymbol{\beta}_j^*(s_j)$: Optimal Buffer allocation vector until $L_j$
$$(j = 1,2,3, \cdots, K_1)$$

$T\left(\beta_j(s_j), \overline{\beta}_j(s_j)\right)$: Throughput of the system given $\boldsymbol{\beta}_j(s_j)$ and $\overline{\boldsymbol{\beta}}_j(s_j)$
$$(j = 1,2,3, \cdots, K_1)$$

$\boldsymbol{B}_j^*(n_j)$: Optimal Buffer-size allocation $L_j$
$$(j=1, 2, 3, \cdots, K_1)$$

$s_j = \sum_{j=1}^{K_1} n_j$: Total buffer-size until $L_j$
$$(j=1, 2, 3, \cdots, K_1)$$

**Conjecture**: Assume that optimal buffer allocation $\boldsymbol{\beta}_j^*(s_j)$
$$\boldsymbol{\beta}_{j+1}^*(s_{j+1}) = T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1} - s_j)\right)$$

Proof: we enumerate four possibilities for $\boldsymbol{\beta}_{j+1}^*(s_j)$, where
$$\beta_j'(s_j) \neq \beta_j^*(s_j), B_j'(n_j) \neq B_j^*(n_j)$$

$$\boldsymbol{\beta}_{j+1}^*(s_{j+1}) = T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1} - s_j)\right) \quad (1)$$

$$\boldsymbol{\beta}_{j+1}^*(s_{j+1}) = T\left(\beta_j^*(s_j), B_{j+1}'(s_{j+1} - s_j)\right) \quad (2)$$

$$\boldsymbol{\beta}_{j+1}^*(s_{j+1}) = T\left(\beta_j'(s_j), B_{j+1}^*(s_{j+1} - s_j)\right) \quad (3)$$

$$\boldsymbol{\beta}_{j+1}^*(s_{j+1}) = T\left(\beta_j'(s_j), B_{j+1}'(s_{j+1} - s_j)\right) \quad (4)$$

Showing that (2),(3) and (4) do not hold, the conjecture is proved.

3.4 Proof of an inconsistent

(i)Proof of not holding (2)

Assume that
$$^\exists B_j^*(s_{j+1} - s_j) \neq B_{j+1}^*(s_{j+1} - s_j), \beta_{j+1}^*(s_{j+1}) = T\left(\beta_j^*(s_j), B_j'(s_{j+1} - s_j)\right) \quad (5)$$

$\beta_{j+1}^*(s_{j+1})$ is the optimal buffer allocation.
$$T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1} - s_j), \beta_{j+1}^*(s_{j+1})\right) < T\left(\beta_{j+1}^*(s_{j+1}), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (6)$$

Using the suppose,
$$T\left(\beta_{j+1}^*(s_{j+1}), \overline{\beta}_{j+1}(s_{j+1})\right) = T\left(\beta_j^*(s_j), B_{j+1}'(s_{j+1} - s_j), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (7)$$

Moreover, $B_{j+1}^*(s_{j+1} - s_j)$ is the optimal buffer allocation of $L_{j+1}$. Using the result of observation,
$$^\forall \overline{\beta}_{j+1} \text{ is}$$

$$T\left(\beta_j^*(s_j), B_{j+1}'(s_{j+1} - s_j), \overline{\beta}_{j+1}(s_{j+1})\right) < T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1} - s_j), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (8)$$

Hence,
$$T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1} - s_j), \beta_{j+1}(s_{j+1})\right) > T\left(\beta_{j+1}^*(s_{j+1}), \beta_{j+1}(s_{j+1})\right) \quad (9)$$

Clearly, (6) and (9) inconsistent, (2) do not hold.

(ii) Proof of not holding (3)

Assume that

$$\beta'_j(s_j) \neq \beta^*_j(s_j), \beta^*_{j+1}(s_{j+1}) = T\left(\beta'_j(s_j), B^*_{j+1}(s_{j+1} - s_j)\right) \tag{10}$$

$\beta^*_{j+1}(s_{j+1})$ is the optimal buffer allocation.

$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1} - s_j), \beta_{j+1}(s_{j+1})\right) < T\left(\beta^*_{j+1}(s_{j+1}), \beta_{j+1}(s_{j+1})\right) \tag{11}$$

Using (10),

$$T\left(\beta^*_{j+1}(s_{j+1}), \overline{B}_{j+1}(s_{j+1})\right) = T\left(\beta'_j(s_j), B^*_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right) \tag{12}$$

$\beta^*_j(s_j)$ is the optimal allocation until $L_j$.

For $^\forall \beta_{j+1}(s_{j+1})$,

$$T\left(\beta'_j(s_j), B^*_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right) < T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right) \tag{13}$$

Hence,

$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right) > T\left(\beta^*_{j+1}(s_{j+1}), \overline{B}_{j+1}(s_{j+1})\right) \tag{14}$$

Clearly, (11) and (14) inconsistent, (3) do not hold.

(iii) Proof of not holding (4)

Assume that $B'_j(s_{j+1} - s_j) \neq B^*_{j+1}(s_{j+1} - s_j)$, $\beta'_j(s_j) \neq \beta^*_j(s_j), \beta^*_{j+1}(s_{j+1}) = T\left(\beta'_j(s_j), B'_{j+1}(s_{j+1} - s_j)\right) \tag{15}$

$\beta^*_{j+1}(s_{j+1})$ is the optimal allocation.

$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right) < T\left(\beta^*_{j+1}(s_{j+1}), \overline{B}_{j+1}(s_{j+1})\right) \tag{16}$$

Here, compare to $T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right)$,

$T\left(\beta^*_j(s_j), B'_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right)$ and

$T\left(\beta'_j(s_j), B'_{j+1}(s_{j+1} - s_j)\right)$

pattern of $T\left(\beta'_j(s_j), B'_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right)$. (17)

We will get the relationship of

$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right) >$$
$$T\left(\beta^*_j(s_j), B'_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right) >$$
$$T\left(\beta'_j(s_j), B'_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right). \tag{18}$$

Moreover,

$$T\left(\beta^*_j(s_j), B'_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right) <$$
$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right) \tag{19}$$

Hence,

$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1} - s_j), \overline{B}_{j+1}(s_{j+1})\right) >$$
$$T\left(\beta^*_j(s_j), \overline{B}^*_{j+1}(s_{j+1} - s_j)\right). \tag{20}$$

Clearly, (16) and (20) inconsistent, (4) do not hold.

The conjecture implies that the optimal allocation until $L_{j+1}$ is obtained by adding the optimal allocation in $L_{j+1}$ to the optimal allocation until $L_j$.

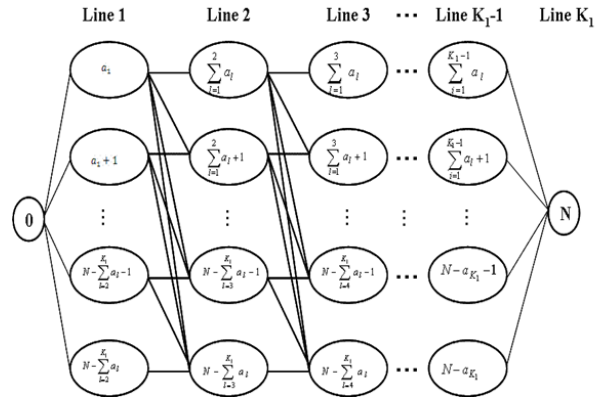This leads to the following dynamic programming algorithm.



Fig 6. Dynamic Programming

4. Dynamic Programming

Based on discussion in this section, here we propose a dynamic programming algorithm to get optimal throughput of the system.

**Notations :**

$0$ :origin

$a_j$ :Buffer-size in $L_j$ $(j = 1,2,3,\cdots, K_1)$

$N$ :Total buffer-size in buffer allocation until $L_{K_1}$
$(j = 1,2,3,\cdots, K_1)$

$S_j : \{s_j | s_j = \sum_{l=1}^{K_1} a_l, \sum_{l=1}^{K_1} a_l + 1, \cdots, N - \sum_{l=1}^{K_1} a_{l+1}\}$
$(j = 1,2,3,\cdots, K_1), (l = 1,2,3,\cdots, K_1)$

Fig 6 denote the dynamic programming. Using the previous section, we must derive the max throughput added by buffer allocation between $L_1$ and $L_j$. We consider the possible buffer size in $L_1$. For the possible buffer size in $L_1$, we can derive buffer allocation. The case of $N$=14 shows that the optimal buffer size in $L_1$ is 5.

As well as, $L_2$, $L_3$ and $L_4$ adapt the process of $L_1$. The thick arrows indicate the optimal route to get the optimal solution.

Holding buffer allocation in $L_1$, we consider the optimal buffer allocation in $L_2$. In this case, the best buffer allocation of $L_1$ is $\beta_1(1,1,2,1)$, the best buffer allocation of $L_2$ is $\beta_{2(1,1,2,1,1,2)}$. As well as, $\beta_3$ and $\beta_4$ is (1,1,2,1,1,2,1,2,2),(1,1,2,1,1,2,1,2,2,1)
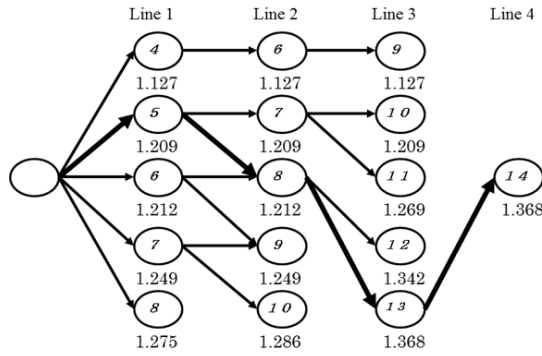
Fig 7.Throughput of $N=14$

Table 3.Buffer Allocation and Optimal Allocation in $N=14$

| Line | Buffer allocation | Optimal allocation |
|---|---|---|
| 1 | $B_1(4)=(1,1,1,1), B_1(5)=(1,1,2,1)$ $B_1(6)=(1,2,2,1), B_1(7)=(1,2,2,2)$ $B_1(8)=(1,2,3,2)$ | $B_1(5)=(1,1,2,1)$ |
| 2 | $B_2(6)=(1,1,1,1,1,1)$ $B_2(7)=(1,1,2,1,1,1)$ $B_2(8)=(1,2,2,1,1,1)$ or $(1,1,2,1,1,2)$ $B_2(9)=(1,2,2,2,1,1,1)$ or $(1,2,2,1,1,2)$ $B_2(10)=(1,2,2,2,1,2)$ | $B_2(8)=(1,1,2,1,1,2)$ |
| 3 | $B_3(9)=(1,1,1,1,1,1,1,1,1)$ $B_3(10)=(1,1,2,1,1,1,1,1,1)$ $B_3(11)=(1,1,2,1,1,1,1,1,2)$ $B_3(12)=(1,1,2,1,1,2,1,1,2)$ $B_3(13)=(1,1,2,1,1,2,1,2,2)$ | $B_3(13)=(1,1,2,1,1,2,1,2,2)$ |
| 4 | $B_4(14)=(1,1,2,1,1,2,1,2,2,1)$ | $B_4(14)=(1,1,2,1,1,2,1,2,2,1)$ |

respectively(See Table 3).

**Remark.** In $L_j$ , $s_1$ signify the range of $a_1$ by $\sum_{l=2}^{K_1} a_l$. For example, for $N = 14$, buffer allocation in $L_1$ represent buffer 5,buffer 8(1),buffer 10(1) and buffer 11 as buffer size in each $L_j$ are all 1.In this case, minimized size in $L_1$ denote $a_1 = 4$. Remaining buffer size $\sum_{l=2}^{4} a_l = 10$ remove $L_1$ regard with $L_{K_1}$. We assume that buffer in $L_2, L_3, \cdots L_{K_1}$ enter in $L_{K_1}$.

Now, maximum $s_1$ with feasible buffer size in $L_1$ given $a_l = 8$ because this case can intend subtract $\sum_{l=2}^{4} a_l$ from N. Therefore, the numbers of feasible buffer allocation in $L_2, L_3$ and $L_4$ shown by $\sum_{l=2}^{4} a_l = 6$. Recall that remaining the number of buffer did not include $L_1$. In other words, $\sum_{l=2}^{4} a_l = 6$ mean that minimum buffer size denoted by feasible buffer allocation from $L_2$ until final line.

For the reasons mentioned above, suppose buffer size in $L_1, L_2, \cdots L_{K_1}$ located all 1, maximum buffer size get $s_1 = 8$ using $N - \sum_{l=2}^{4} a_l$. Therefore, the range allocation size of $L_1$ will find between $s_1 = 4$ and $s_1 = 8$.

Next, using the buffer allocation size ($s_1$), we note buffer allocation by enumeration with simulation result (i.e. maximum $T\left(B_j(n_j)|\bar{B}_j(n_j)\right)$ and buffer allocation) and hold the buffer allocation state to derive the optimal buffer allocation in $L_{K_1}$. As well as $L_1, L_2$ consists of buffer 2 and buffer 6. Notice that minimum feasible buffer size in $L_2$ is not

$a_2 = 2$ but mixing $a_1 = 4$ and $a_2 = 2$. $a_1 = 4$ , $a_2 = 2$ signify minimum $\sum_{l=1}^{2} a_l$ (min $s_2$ is 6) and then max $s_2$ is given by remaining the number of buffer in $L_3, L_4$ did not include $L_2$ (i.e. max $s_2$ is 10). $L_3, L_4$ can derive the same as $L_1$ and $L_2$ method.

**The algorithm for optimal buffer-size allocation:**
Step 0) Set $j=1$.

Step1-1) Calculate $T\left(B_j(n_j),\bar{B}_j(n_j)\right)$ for all $B_j(n_j)$'s to derive

Step1-2) Set $\beta_j^*(n_j) = B_j^*(n_j)$ for each feasible $n_j$

Step2-1) For each feasible $s_j$, keep $\beta_j^*(s_j)$ unchanged and calculate $T\left(B_{j+1}(s_{j+1} - s_j), \bar{B}_{j+1}(s_{j+1} - s_j)\right)$ for all $B_{j+1}(s_{j+1} - s_j)$'s to derive $B_{j+1}^*(s_{j+1} - s_j)$ for each feasible $s_{j+1}$.

Step2-2) For each feasible $s_{j+1}$, select optimal $\beta_{j+1}^*(s_{j+1}) = T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1} - s_j)\right)$ from all feasible combina tions of $s_j$ and $s_{j+1} - s_j$.

Step 3) If $j=K_1$, $\beta_{j+1}^*(s_{j+1})$ is the optimal buffer-size allocation vector. Otherwise set $j=j+1$ and go back to Step 2-1).

## 5. Validation
In this section, we calculate Percentage Reduction (PR) in all allocation type with total buffer size. It defines Total Allocation (TA) and SA with search for allocation.
We have

$$PR = 1 - \frac{SA}{TA}$$

For $N=14$, we search for 210 allocation types. Buffer allocation type was increased by the number of total buffer size. Hence, increasing buffer allocation increased the magnitude of PR and allocation type. Consequently, independent from the magnitude of buffer size, an algorithm applied for holding the buffer allocation.

Table 4. Search Result

| Total buffer size | Total buffer allocation type | Search for buffer allocation | PR |
|---|---|---|---|
| 12 | 55 | 27 | 50.9% |
| 13 | 220 | 83 | 62.2% |
| 14 | 715 | 210 | 70.6% |
| 15 | 2002 | 459 | 77.1% |

## 6. Conclusion

In this paper, we discussed optimal buffer-size allocation problem in assembly-like queueing systems to propose a dynamic programming algorithm that maximizes throughput of the system. Different from previous studies on heuristic policies, we proposed a systematic algorithm.

However, future research is necessary to find theoretical support for the observation in Section 3.

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