Decentralized Controller Design Using Dynamic Output Feedback

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Abstract— To control the large scale systems is important. In this paper, a multi variable non-linear system (two inverted pendulum coupled by a spring) is output feedback linearized and the system is generalized in two subsystems and decentralized dynamic output feedback basis on Lyapunov equation is applied. Using this model, the large scale system can be formulated, designed and generalized to be controlled.

***Keywords*—** Decentralized Control, Dynamic Output Feedback, Large Scale System

I. INTRODUCTION

Power systems and multimachine systems are modeled as large nonlinear highly structured systems [1]. Despite the importance and potentials of large-scale systems, it seems that impact of the research in this area is not as great as it could be. There are indeed many successful applications of large-scale systems control, for example to electrical power systems [2]. However, these applications are mainly developed by domain experts. All applications in this area are "large-scale", i.e. the number of state variables is very big and special knowledge is normally required for the formulation of the problem. In the general control communities, due to lack of simple meaningful examples, the interests in this area are not matched with its importance and potentials [3]. Conventional linear control for large-scale systems is limited since it can only deal with small disturbances about an operating point. Since differential geometric tools were introduced to nonlinear control system design, various stabilizing control results based on nonlinear power system models have been obtained for single machine systems and for multimachine systems. Two important issues for power systems control are robustness and a decentralized structure. However, other method such as Genetic Algorithm and Neural Network Method are used to model multimachine systems [4]. The robustness issue arises to deal with sources of uncertainties which mainly come from the varying network topology and the dynamic variation of the load. Since physical limitation on the system structure makes information transfer among subsystems unfeasible, decentralized controllers for multimachine systems must be used [1]. Therefore, decentralized control is considered as an effective method to deal with large-scale interconnected systems. In addition, it is often used to utilize the system structural characteristics, such as symmetric structure [4], cascaded structure [5] or similar structure [6] to study special large-scale systems as a first step toward general large-scale systems [7]. Decentralized dynamic output feedback basis on Lyapunov equation is an effective method to study the stability of a complex system [8]. Also, it can be used to analyze the behavior of design process and stability analysis of the system [9]-[10]. A double system pendulums coupled by a spring (Fig 1) was used in [3] to demonstrate some important theoretical results achieved in decentralized control. This system is used in this paper to be studied using dynamic output feedback. A character for modeling this system is that by simply adding more pendulums and springs to the existing system, this can be extended to a system of n-inverted-pendulums coupled by (n-1)-springs. Such a system can be observed as an attractive example for the current research on "characterization of problems in decentralized control" [3].

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II. NOTATION AND PRELIMINARIES [11]

For a given matrix A, let A^T denote its transport; let $\overline{\sigma}(A)$ denotes the maximum singular values of A; when A is real symmetric, let $\underline{\lambda}(A)$ and $\overline{\lambda}(A)$ denote its minimum and maximum eigenvalues, respectively; let $A > 0$ denotes that A is positive define. Let L denotes Lipschitz constant of the function $F(x)$ in its domain of definition; also, let $\|.\|$ denote the Euclidean norm or its induced norm.

Consider the following two systems:

$$(\Sigma_I) \qquad \begin{aligned} \dot{x} &= f_1(x) + g_1(x)u \\ y_1 &= h_1(x) \end{aligned}$$

$$(\Sigma_{II}) \qquad \begin{aligned}\dot{\hat{x}} &= f_2(\hat{x}) + g_2(\hat{x})u \\ y_2 &= h_2(\hat{x})\end{aligned}$$

Where $x, \hat{x} \in \mathbb{R}^n$, $y_1, y_2 \in \mathbb{R}^l$ and $u \in \mathbb{R}^m$ are the state vectors, outputs, and inputs of systems.

Definition 2.1. Σ_I is said to be similar to . Σ_{II} in the domain E if there exists a diffeomorphism $T: x \to \hat{x}$ defined in E such that in the coordinate $\hat{x}$ defined by T, the system Σ_I possesses the same form as Σ_{II}. In this case, $T(x)$ is called a similarity transformation from Σ_I to Σ_{II}.

Remark 2.1. Similarity between systems is an equivalence relationship. That is, similarity possesses the properties of reflectivity, symmetry and transitivity; it is an extension of equivalence between linear systems.

Definition 2.2. the system Σ_I is said to be output feedback linearizable in the domain E if there exists a diffeomorphism $T: x \to z$, $\alpha(y) \in \mathbb{R}^m$ and a nonsingular matrix $\beta(y) \in \mathbb{R}^{m\times m}$ such that in the coordinate z defined by T, the closed-loop system resulting from the input $u = \alpha(y) + \beta(y)v$ to Σ_I is described by

$$\dot{z} = Az + Bv$$
$$y = Cz$$

With the realization (A, B, C) both controllable and observable.

Remark 2.2. It should be noticed that output feedback linearizability defined above does not imply static output feedback stabilizability. However, the fact that a system is output feedback linearizable. It should be mentioned that static output feedback stabilizability is a very strong condition and it remains an open problem even for linear systems.

Lemma 2.1. Suppose that Σ_I is similar to Σ_{II} in the domain E. Then, Σ_I is output feedback linearizable if and only if Σ_I is output feedback linearizable.

Proof. Necessity, by output feedback linearization of Σ_I, it follows that there exists a similarity transformation $T: x \to z$, $\alpha(y) \in \mathbb{R}^m$ and a nonsingular matrix $\beta(y) \in \mathbb{R}^{m\times m}$ such that in the coordinate z defined by T_1, the closed-loop system

$$\dot{x} = f_I(x) + g_i(x)[\alpha(y_I) + \beta(y_I)v] \qquad (2)$$
$$y_I = h_I(x) \qquad (3)$$

Has the following form

$$\dot{z} = Az + Bv \qquad (4)$$
$$y = Cz \qquad (5)$$

It is obvious that the systems (2)-(3) is similar to (4)-(5) with similarity transformation T_1. Now, suppose that Σ_I is similar to Σ_{II} with similarity transformation T_2. Then, it is observed that the system

$$\dot{\hat{x}} = f_{II}(\hat{x}) + g_{II}(\hat{x})[\alpha(\hat{y}_{II}) + \beta(\hat{y}_{II})v] \qquad (6)$$
$$y_{II} = h_{II}(\hat{x}) \qquad (7)$$

Is similar to (2)-(3) with similarity transformation T_2^{-1}. By the properties of similarity, it follows that (6)-(7) is similar to (4)-(5) with similarity transformation T_2^{-1} to T_1. Therefore, for Σ_{II}, there exists a composition of transformation given by T_2^{-1} to T_1 and output feedback $u = \alpha(\hat{y}_{II}) + \beta(\hat{y}_{II})v$ such that the resulting closed-loop system is linearizable.

Sufficiently, it may be obtained directly from the necessity proof and the symmetry property of the similarity transformation. Hence, the result follows.

III. System discription

Consider a nonlinear large-scale interconnected system described by

$$\dot{x}_i = f_i(x_i) + g_i(x_i)[u_i + \Delta\Psi_i(x_i)] + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(x_j) + \Delta H_i(x) \qquad (8)$$
$$y_i = h_i(x_i) \qquad i = 1, 2, \ldots, N \qquad (9)$$

Here $x_i \in \Omega_i \in \mathbb{R}^n$($\Omega_i$ is a neighborhood of $x_i = 0$), $u_i, y_i \in \mathbb{R}^m$ are the state vector, input and output vector of the ith subsystems, respectively; $f_i(x_i)$, $g_i(x_i)$ are both smooth vectors, h_i is a smooth function in Ω_i, $\Delta\Psi_i(x_i)$ is the matched uncertainty of the ith isolated subsystem; $\sum_{j=1}^{N} H_{ij}(x_j)$, j=i is the known interconnection, the uncertain interconnection $\Delta H_i(x)$ includes all unmatched uncertainties, and they are all continuous in their arguments. Without loss of generality, it is supposed that $f_i(0) = 0$ and $h_i(0) = 0$. Also, we write

$$x = col(x_1, x_2, .., x_N) \in \Omega_1 \times \Omega_2 \times \ldots \Omega_N = \Omega$$

Definition 1. consider the system (8)-(9). The systems

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i \qquad (10)$$
$$y_i = h_i(x_i) \qquad i = 1, 2, \ldots, N \qquad (11)$$

are called nominal subsystems of the system (8)-(9); the systems

$$\dot{x}_i = f_i(x_i) + g_i(x_i)[u_i + \Delta\Psi_i(x_i)] \qquad (12)$$
$$y_i = h_i(x_i) \qquad i = 1, 2, \ldots, N \qquad (13)$$

are called isolated subsystems of the system (8)-(9).

Definition 2. The system (8)-(9) is said to be a similar interconnected large-scale system or to posses a similar structure if all its nominal subsystems are similar to one another.

Remark 1. It should be noticed from remark 2.1 that there exists one system such that each nominal subsystem of the system (8)-(9) is similar to the system, if (8)-(9) is a similar interconnected large-scale system.

Assumption A1. The system (8)-(9) possesses a similar structure and there exists one output feedback linearizable nominal subsystem.

It should be noticed from Definition 2.2 that Assumption A1 does not imply output feedback linearizability of the nominal subsystem (10)-(11) of the system (8)-(9). In fact, we do not require that the nominal subsystem (10)-(11) of the system (8)-(9) is output feedback stabilizable in this paper; on the other hand, the nominal subsystem is required to be linear and output feedback stabilizable in ref 8.

By Lemma 2.1, it is observed that, under Assumption A1, all nominal subsystems of the system (8)-(9) are similar. Therefore, there exists diffeomorphisms $T_i: x_i \to z_i$ and output feedback

$$u_i = \alpha(y_I) + \beta(y_I)v \qquad (14)$$

In Ω_i, for $i = 1, 2, \dots, N$ such that, in the new coordinate $z = \mathrm{col}(z_1, z_2, \dots, z_N)$, the system (8)-(9) is described by

$$\dot{z}_i = Az_i + B[v_i + \Delta\Phi_i(z_i)] + \sum_{\substack{j=1\\ j\neq i}}^{N} M_{ij}(z_i, z_j) + \Delta M_i(z) \quad (15)$$

$$y_i = Cz_i, \qquad i = 1, 2, \dots, N \quad (16)$$

where the realization (A, B, C) is controllable and observable and where

$$\Delta\Phi_i(z_i) = [\beta_i^{-1}(y_i)\Delta\Psi_i(x_i)]_{x_i = T_i^{-1}(z_i)} \quad (17)$$

$$\Delta M_i(z) = \left[\frac{\partial T_i(x_i)}{\partial x_i}\right]_{x_i = T_i^{-1}(z_i)} . \Delta H_i(T^{-1}(z)) \quad (18)$$

$$M_{ij}(z_i, z_j) = \left[\frac{\partial T_i(x_i)}{\partial x_i}\right]_{x_i = T_i^{-1}(z_i)} . H_{ij}\left(T_j^{-1}(z_j)\right), i \neq j \quad (19)$$

with $i, j = 1,2, \dots, N$ and

$$z = T(x) = \mathrm{col}(T_1(x_1), T_2(x_2), \dots, T_N(x_N))$$

Assumption A2. $M_{ij}(z_i, z_j), i \neq j$ is Lipschitz in $T_i(\Omega_i) \times T_j(\Omega_j)$ with Lipschitz constants $L_{M_{ij}}^{i}$ and $L_{M_{ij}}^{j}$. That is, for any $z_i, \tilde{z}_i \in T_i(\Omega_i)$ and $z_j, \tilde{z}_j \in T_j(\Omega_j)$,

$$\left\|M_{ij}(z_i, z_j) - M_{ij}(\tilde{z}_i, \tilde{z}_j)\right\| \leq L_{M_{ij}}^{i} \|z_i - \tilde{z}_i\| + L_{M_{ij}}^{j} \|z_j - \tilde{z}_j\|, \quad j \neq i$$

and $M_{ij}(z_i, z_j)$ has the following description:

$$M_{ij}(z_i, z_j) = \Pi_{ij}(y_i, y_j) z_j, \quad j \neq i \quad (20)$$

with $\Pi_{ij} \in \mathbb{R}^{n\times n}$ for all $i = 1, 2, \dots, N$.

Assumption A3. There exist known continuous functions $\rho_i(.)$ and $\gamma_i(.)$, defined in their domains of definition, such that, for i=1, 2, …,N,

$$\|\Delta\Phi_i(z_i)\| \leq \rho_i(\|y_i\|)\|y_i\|$$

$$\|\Delta M_i(z)\| \leq \gamma_i(y)\|z\|$$

IV. Dynamic Output feedback Controller Design

Consider the system (15)-(16). From the controllability and observability of the realization (A, B, C), it follows that there exist K, L such that, for any $Q > 0$ and $S > 0$, the following Lyapunov equations:

$$(A - BK)^T P + P(A - BK) = -Q \quad (21)$$

$$(A - LC)^T R + R(A - LC) = -S \quad (22)$$

have unique solutions $P > 0$ and $R > 0$, respectively.

Assumption A4. There exists matrix F such that $B^T P = FC$, with P defined by (21).

Consider the system (8)-(9). Construct the controller described by

$$\dot{\hat{x}}_i = f_i(\hat{x}_i) + g_i(\hat{x}_i)u_i + \left[\frac{\partial T_i(\hat{x}_i)}{\partial \hat{x}_i}\right]^{-1} L[y_i - h_i(\hat{x}_i)] + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(\hat{x}_j) \quad (23)$$

$$y_i = h_i(x_i) \quad (24)$$

$$u_i = \alpha_i(y_i) + \beta_i(y_i)[-KT_i(\hat{x}_i) + \eta_i(\eta_i(y_i))], \quad i = 1,2, \dots, N \quad (25)$$

where K satisfies (21) and $\eta_i(.)$ is defined by, for i=1,2,…N,

$$\eta_i(y_i) = \begin{cases} -\left[\frac{Fy_i}{\|Fy_i\|}\right]\rho_i(\|y_i\|)\|y_i\|, & Fy_i \neq 0 \\ 0, & Fy_i = 0 \end{cases} \quad (26)$$

with $\rho_i(\|y_i\|)$, for i=1,2,…N and F defined respectively by assumptions A3 and A4. Now, we have the following result.

Theorem 1. Under assumptions A1-A4, the system (8)-(9) is stabilized by the controller (23)-(25) if there exists a neighborhood about the origin $\Omega' \subseteq \Omega$ such that $W + W^T > 0$ in $\Omega' \backslash \{0\}$, where $W = [w_{ij}]_{2N\times 2N}$ is defined by

$w_{ij} = \underline{\lambda}(Q) - 2\overline{\lambda}(P)\gamma_i(y)$,
$1 \leq i \leq N, i = j$
$w_{ij} = \underline{\lambda}(S) - 2\overline{\lambda}(R)\sum_{k=N+1,k\neq i}^{2N} L_{M_{(i-k)(k-N)}}^{i-N}$,
$N + 1 \leq i \leq 2N, i = j$

$w_{ij} = -2\left(\overline{\sigma}\left(P\Pi_{ij}(y_i, y_j)\right) + \overline{\lambda}(P)\gamma_i(y)\right)$
$1 \leq i, j \leq N, i \neq j$
$w_{ij} = -2\overline{\lambda}(R)\, L_{M_{(i-N)(j-N)}}^{j-N}$,
$N + 1 \leq i, j \leq 2N, i \neq j$
$w_{ij} = -\overline{\sigma}(RB)\overline{\sigma}(C)\rho_i(\|y_i\|) - \overline{\sigma}(PBK) - \overline{\lambda}(R)\gamma_i(y)$,
$j - i = N, 1 \leq i \leq N$
$w_{ij} = -\overline{\sigma}(RB)\overline{\sigma}(C)\rho_j(\|y_i\|) - \overline{\sigma}(PBK) - \overline{\lambda}(R)\gamma_j(y)$
$i - j = N, 1 \leq j \leq N$
$w_{ij} = -\underline{\lambda}(R)\gamma_{j-N}(y)$,
$j - i \neq N, 1 \leq i \leq N, \quad N \leq j \leq 2N$
$w_{ij} = -\underline{\lambda}(R)\gamma_{i-N}(y)$,
$j - i \neq N, 1 \leq j \leq N, \quad N \leq i \leq 2N$

Where P, Q, R, S are defined by (21)-(22) [7].

Proof: It is obvious that the closed-loop system obtained by the controller (23)-(25) to the system (8)-(9) is described by

$$\dot{x}_i = f_i(x_i) + g_i(x_i)\{\alpha_i(y_i) + \beta_i(y_i)[-KT_i(\hat{x}_i) + \eta_i(y_i)] + \Delta\psi_i(x_i)\} + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(x_j) + \Delta H_i(x), \quad (27)$$

$$\dot{\hat{x}}_i = f_i(x_i) + g_i(x_i)\{\alpha_i(y_i) + \beta_i(y_i)[-KT_i(\hat{x}_i) + \eta_i(y_i)]\} + [\partial T_i(\hat{x}_i)/\partial\hat{x}_i]^{-1} L[y_i - h_i(\hat{x}_i)] + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(x_j), \quad (28)$$

$$y_i = h_i(x_i), \qquad i = 1, 2, \dots, N, \quad (29)$$

where K, L are denoted by (21)-(22) and $\eta_i(.)$ is denoted by (26).

For the system (27)-(29), we construct a Lyapunov function candidate as

$$V = \sum_{i=1}^{N}\left\{\left(T_i(x_i)\right)^T P T_i(x_i) + [T_i(x_i) - T_i(\hat{x}_i)]^T R [T_i(x_i) - T_i(\hat{x}_i)]\right\} \quad (30)$$

where P, S are defined by (21)-(22). It follows from Assumption A1 that, in the new coordinates $z, \hat{z}$, system (27)-(29) is described by

$$\dot{z}_i = Az_i - BK\hat{z}_i + B[\eta_i(y_i) + \Delta\Phi_i(z_i)] + \sum_{\substack{j=1\\ j\neq i}}^{N} M_{ij}(z_i, z_j) + \Delta M_i(z), \quad (31)$$

$$\dot{\hat{z}}_i = (A - BK)\hat{z}_i + B\eta_i(y_i) + L(y_i - C\hat{z}_i) + \sum_{\substack{j=1\\ j\neq i}}^{N} M_{ij}(z_i, z_j), \quad (32)$$

$$y_i = Cz_i \qquad i = 1, 2, \dots, N \quad (33)$$

where the realization (A, B, C) is the same as (15)-(16), and where $\Delta\Phi_i, \Delta M_i, M_{ij}$ are defined by (17)-(19). Let

$$e_i : T_i(x_i) - T_i(\hat{x}_i) = z_i - \hat{z}_i \qquad \text{for } i = 1, 2, \dots, N.$$

We have, for $i = 1, 2, \dots, N$,

$$\begin{bmatrix}\dot{z}_i\\ \dot{e}_i\end{bmatrix} = \begin{bmatrix}A - BK & BK\\ 0 & A - LC\end{bmatrix}\begin{bmatrix}z_i\\ e_i\end{bmatrix} + \begin{bmatrix}B[\Delta\Phi_i(z_i) + \eta(y_i)]\\ B\Delta\Phi_i(z_i)\end{bmatrix} + \begin{bmatrix}\sum_{j=1, J\neq i}^{N} M_{ij}(z_i, z_j) + \Delta M_i(z)\\ \sum_{j=1, J\neq i}^{N}[M_{ij}(z_i, z_j) - M_{ij}(\hat{z}_i, \hat{z}_j) + \Delta M_i(z)]\end{bmatrix} \quad (34)$$

The time derivative of V along the trajectories of the system (27)-(29) is given by

$$\dot{V}|_{(27)-(29)} = -\sum_{i=1}^{N}[z_i{}^T, e_i{}^T]\operatorname{diag}(Q, S)\begin{bmatrix}z_i\\ e_i\end{bmatrix} + 2\sum_{i=1}^{N}[z_i{}^T, e_i{}^T]\operatorname{diag}(P, R) \times \left\{\begin{bmatrix}B[\Delta\Phi_i(z_i) + \eta_i(y_i + Ke_i)]\\ B\Delta\Phi_i(z_i)\end{bmatrix} + \begin{bmatrix}\sum_{j=1, J\neq i}^{N} M_{ij}(z_i, z_j) + \Delta M_i(z)\\ \sum_{j=1, J\neq i}^{N}[M_{ij}(z_i, z_j) - M_{ij}(\hat{z}_i, \hat{z}_j) + \Delta M_i(z)]\end{bmatrix}\right\} \quad (35)$$

By the structure (26) of $\eta_i(y_i)$ and Assumption A4, it is observed that:

(i) if $Fy_i = 0$, then for $i = 1, 2, \dots, N$

$$z_i{}^T PB[\Delta\Phi(z_i) + \eta_i(y_i)] = (FCz_i)^T\Delta\Phi(z_i) = (Fy_i)^T\Delta\Phi_i(y_i) = 0;$$

(ii) if $Fy_i \neq 0$, then for $i = 1, 2, \dots, N$

$$\begin{aligned} z_i{}^T PB[\Delta\Phi(z_i) + \eta_i(y_i)] &\leq \|z_I^T(FC)^T\|\rho_i(\|y_i\|)\|y_i\| - \left[\frac{z_I^T(FC)^T Fy_i}{\|Fy_i\|}\right]\rho_i(\|y_i\|)\|y_i\| \\ &\leq \|FCz_i\|\rho_i(\|y_i\|)\|y_i\| - \left[\frac{(FCz_i)^T FCz_i}{\|FCz_i\|}\right]\rho_i(\|y_i\|)\|y_i\| \\ &= 0 \end{aligned}$$

Therefore, for $i = 1, 2, \dots, N$,

$$z_i^T PB[\Delta\Phi_i(z_i) + \eta_i(y_i)] \leq 0 \quad (36)$$

Then, by Assumption A3, it follows that, for $i = 1, 2, \dots, N$,

$$\begin{aligned} &e_i^T RB\Delta\Phi_i(z_i) + z_i^T PBKe_i \\ &\leq \|e_i\|\|RB\|\rho_i(\|y_i\|)\|y_i\| + \|z_i^T PBKe_i\| \\ &\leq \rho_i(\|y_i\|)\bar{\sigma}(RB)\bar{\sigma}(C) + \bar{\sigma}(PBK)\|z_i\|\|e_i\| \end{aligned} \quad (37)$$

From Assumptions A2-A3 and fact that

$$\|z\| \leq \|z_1\| + \|z_2\| + \dots + \|z_N\|$$

It is observed that

$$\begin{aligned} &z_i^T P\left[\sum_{\substack{j=1\\ j\neq i}}^{N} M_{ij}(z_i, z_j) + \Delta M_i(z)\right] \\ &= \sum_{\substack{j=1\\ j\neq i}}^{N} z_i^T P\Pi_{ij}(y_i, y_j)z_j + z_i^T P\Delta M_i(z) \\ &\leq \sum_{\substack{j=1\\ j\neq i}}^{N} \bar{\sigma}\left(P\Pi_{ij}(y_i, y_j)\right)\|z_i\|\|z_j\| + \bar{\lambda}(P)\gamma_i(y)\|z_i\|\|z_j\| \\ &\leq \sum_{\substack{j=1\\ j\neq i}}^{N} \bar{\sigma}\left(P\Pi_{ij}(y_i, y_j)\right)\|z_i\|\|z_j\| + \sum_{j=1}^{N}\bar{\lambda}(P)\gamma_i(y)\|z_i\|\|z_j\| \end{aligned} \quad (38)$$

Also, Assumptions A2-A3 give

$$\begin{aligned} &e_i^T R\left[\sum_{\substack{j=1\\ j\neq i}}^{N}[M_{ij}(z_i, z_j) - M_{ij}(\hat{z}_i, \hat{z}_j)] + \Delta M_i(z)\right] \\ &\leq \sum_{\substack{j=1\\ j\neq i}}^{N} \bar{\lambda}(R)\|e_i\|\|M_{ij}(z_i, z_j) - M_{ij}(\hat{z}_i, \hat{z}_j)\| + \bar{\lambda}(R)\gamma_i(y)\|e_i\|\|z\| \\ &\sum_{\substack{j=1\\ j\neq i}}^{N} \bar{\lambda}(R)\|e_i\|\left[L_{M_{ij}}^i\|z_i - \hat{z}_i\| + L_{M_{ij}}^i\|z_j - \hat{z}_j\|\right] + \sum_{i=1}^{N}\bar{\lambda}(R)\gamma_i(y)\|e_i\|\|z_j\| \\ &= \bar{\lambda}(R)\left[+\sum_{\substack{k=1\\ k\neq i}}^{N} L_{M_{ik}}^i\right]\|e_i\|^2 + \bar{\lambda}(R)\sum_{\substack{j=1\\ j\neq i}}^{N} L_{M_{ij}}^j\|e_i\|\|e_j\| \\ &+ \sum_{j=1}^{N}\bar{\lambda}(R)\gamma_i(y)\|e_i\|\|z_j\| \end{aligned} \quad (39)$$

Substituting (326)-(39) into (35) yields

$$\begin{aligned} &\dot{V}\big|_{(27)-(29)} \\ &\leq -\sum_{i=1}^{N}\left[\underline{\lambda}(Q)\|z_i\|^2 + \underline{\lambda}(S)\|e_i\|^2\right] \\ &+2\sum_{i=1}^{N}[\rho_i(y_i)\bar{\sigma}(RB)\bar{\sigma}(C) + \bar{\sigma}(PBK)]\|z_i\|\|e_i\| \\ &+2\sum_{i=1}^{N}\left\{\sum_{\substack{j=1\\ j\neq i}}^{N}\left[\bar{\sigma}\left(P\Pi_{ij}(y_i, y_j) + \bar{\lambda}(P)\gamma_i\right)\|z_i\|\|z_j\| + \bar{\lambda}(P)\gamma_i(y)\|z_i\|^2\right]\right\} \\ &+2\sum_{i=1}^{N}\bar{\lambda}(R)\left[\sum_{\substack{k=1\\ k\neq i}}^{N} L_{M_{ik}}^i\right]\|e_i\|^2 + 2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\bar{\lambda}(R)L_{M_{ij}}^i\|e_i\|\|e_j\| \\ &+2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\bar{\lambda}(R)\gamma_i(y)\|z_i\|\|e_j\| \\ &= -(1/2)Y^T(W^T + W)Y \end{aligned} \quad (40)$$

where

$$Y = (\|z_1\|, \|z_2\|, \dots \|z_N\|, \|e_1\|, \|e_1\|, \dots, \|e_N\|)^T$$

It is obvious that $Y = 0$ if and only if $x_i = \hat{x}_i = 0$ for $i = 1, 2, \dots, N$. then, from the positive definiteness of $W^T + W$, it follows that the system (27)-(29) is asymptotically stable at origin. Hence, the result follows.

It should be emphasized that the proof of Theorem 4. 1 is constructive. The robustness is enhanced greatly compared with existing results in which all the uncertainties are estimated or not considered in the control design.

In addition, from the continuity of $\rho_i(.)$, it is observed that u_i is continuous if F is nonsingular. From the proof above, it should be emphasized that Assumption A4 is unnecessary if the matched uncertainties $\Delta\Psi_i$, with $i = 1, 2, \dots, N$, do not appear in (8).

Corollary 4. 1. Under the conditions of Theorem 4. 1, (23) is an asymptotic observer of the system (8)-(9).

In other words, under certain conditions, the system (8)-(9) is stabilized by the controller (25), based on the system output and estimated states given by (23).

V. Modeling

A system of two inverted pendulum coupled by a spring is shown in figure1. The variables of the system are:

θ_i: angular displace ment of pendulum I (i=1, 2)

τ_i: torque input generated by the actuator for pendulum I (i=1, 2)

F: spring force

ϕ: angular of the spring to the earth

and the constants are:

m_i: mass of pendulum

L: distance of two pendulums

κ: spring constant

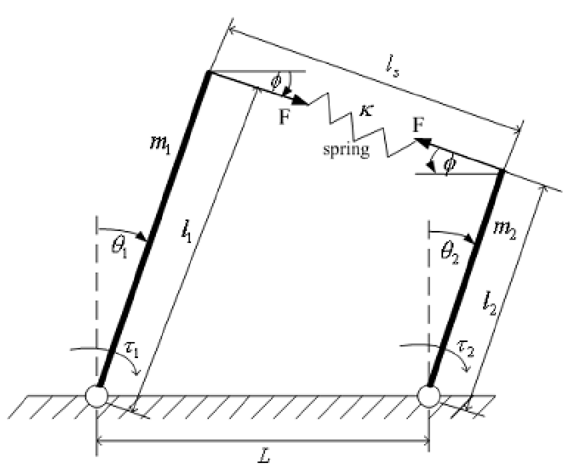

Fig 1. Two inverted pendulum coupled by spring

The mass of each pendulum is uniformly distributed. The length of spring is chosen so that F=0 when $\theta_1 = \theta_2 = 0$, which implies that $(\theta_1\ \dot{\theta}_1\ \theta_2\ \dot{\theta}_2)^T = 0$ is an equilibrium of the system if $\tau_i = 0$. For simplicity, we assume that the mass of spring is zero.

The dynamic equations for the system of fig.1 are given as

$$[m_1(l_1)^2/3]\ddot{\theta}_1 = \tau_1 + m_1 g(l_1/2)\sin\theta_1 + l_1 F\cos(\theta_1 - \phi) \quad (41)$$

$$[m_1(l_2)^2/3]\ddot{\theta}_2 = \tau_2 + m_2 g(l_2/2)\sin\theta_2 + l_2 F\cos(\theta_2 - \phi) \quad (42)$$

where $g = 9.8\mathrm{m/s^2}$ is the constant of gravity and

$$F = \kappa\left(l_s - [L^2 + (l_2 - l_1)^2]^{1/2}\right) \quad (43)$$

$$l_s = [(L + l_2\sin\theta_2 - l_1\sin\theta_1)^2 + (l_2\cos\theta_2 - l_1\cos\theta_1)^2]^{1/2} \quad (44)$$

$$\phi = \tan^{-1} {l_1\cos\theta_1 - l_2\cos\theta_2}/{L + l_1\cos\theta_1 - l_2\cos\theta_2} \quad (45)$$

The following variables are used: $l_1 = 1\mathrm{m}$ ، $l_2 = 0.8\mathrm{m}$ ، $m_1 = 1\mathrm{kg}$ ، $m_2 = 0.8\mathrm{kg}$ ، $L = 1.2\mathrm{m}$ and $\kappa = 0.04\mathrm{N/m}$ [3].

VI. Controller design

Now, the system of two inverted pendulum coupled by spring with governing equations (1)-(2) is set decentralized nonlinear control using dynamic output feedback. First, the state equations can be written considering the definition of state variables:

$$x_1 = \theta_1, x_2 = \dot{\theta}_1, x_3 = \theta_2, x_4 = \dot{\theta}_2 \quad (46)$$

then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 14.7\sin x_1 \\ 0 \\ 18.37\sin x_3 \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 \\ 3 & 0 \\ 0 & 0 \\ 0 & 5.86 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 3F\cos(x_1 - \phi) \\ 0 \\ -4.6875F\cos(x_3 - \phi) \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} \quad (47)$$

Then we classify the states in two classes

$$z_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad z_2 = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \tag{48}$$

So, the equation (47) is converted to the following equations:

$$\dot{z}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_1 + \begin{bmatrix} 0 \\ 14.70\sin(x_1) \end{bmatrix} + \begin{bmatrix} 0 \\ 3.0 \end{bmatrix} \tau_1 + \begin{bmatrix} 0 \\ 3F\cos(x_1 - \phi) \end{bmatrix}$$

$$y_1 = [1 \quad 1] z_1 \tag{49}$$

$$\dot{z}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_2 + \begin{bmatrix} 0 \\ 18.37\sin(x_3) \end{bmatrix} + \begin{bmatrix} 0 \\ 5.86 \end{bmatrix} \tau_2 + \begin{bmatrix} 0 \\ 3F\cos(x_3 - \phi) \end{bmatrix}$$

$$y_2 = [1 \quad 1] z_2 \tag{50}$$

Considering the theorem 4.1, we study the assumption A1 to A4. To define the equivalent terms of system (3) in pendulum system, we have

$$f_1(z_1) = \begin{bmatrix} z_2 \\ 14.70\sin(x_1) \end{bmatrix}_{x_1 = [1 \quad 0] z_1}, \quad g_1(z_1) = \begin{bmatrix} 0 \\ 3.0 \end{bmatrix},$$

$$h_1(z_1) = [1 \quad 1] z_1 \tag{51}$$

$$\Delta\Psi_i(z_1) = 0, \ \sum_{\substack{j=1 \\ j \neq i}}^{2} H_{ij}(z_j) = 0,$$

$$\Delta H_1(z) = \begin{bmatrix} 0 \\ 3F\cos(x_1 - \phi) \end{bmatrix}, \tag{52}$$

Similarity for subsystem 2, we have

$$f_2(z_2) = \begin{bmatrix} z_2 \\ 18.37\sin(x_3) \end{bmatrix}_{x_3 = [1 \quad 0] z_2}, \quad g_2(z_2) = \begin{bmatrix} 0 \\ 5.86 \end{bmatrix},$$

$$h_2(z_2) = [1 \quad 1] z_2 \tag{53}$$

$$\Delta\Psi_i(z_2) = 0,$$

$$\sum_{\substack{j=1 \\ j \neq i}}^{2} H_{ij}(z_j) = 0, \quad \Delta H_2(z) = \begin{bmatrix} 0 \\ 3F\cos(x_3 - \phi) \end{bmatrix} \tag{54}$$

It is clear that the system has the same structure and can get output feedback linearization. So, the first assumption is achieved. System is suitable for feedback linearization, so no transformation is needed to a new system. The matrices of equation (3) considering equations (20) to (23) will be:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C = [1 \quad 1] \tag{55}$$

$$\Delta\Phi_i(z_i) = 0, \ M_{ij}(z_i, z_j) = 0, \ \Delta M_i(z) = \Delta H_i(z) \tag{56}$$

The second assumption is confirmed by becoming zero of interconnection terms. Control inputs (applied torque) is achieved by equation (14). Consider as $\rho_i(\|y_i\|)$ is zero, so $\eta_i(y_i) = 0$.

$$\tau_1 = (-Kz_1 + 14.7\sin(x_1))/3 \tag{57}$$

$$\tau_2 = -(Kz_2 + 18.37\sin(x_3))/5.86 \tag{58}$$

in which K is defined by equation (7). Also, consider that K must be chosen in order to have asymptotic stability for matrix $A - BK$. So, $K = [3 \quad 6]$ is defined. In Lyapunov equation, by considering the fourth assumption and choosing $S = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, the response of equations are as follows:

$$P = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}, \ R = \begin{bmatrix} 0.0859 & 0.0312 \\ 0.0312 & 0.0859 \end{bmatrix} \tag{59}$$

To satisfy assumption A4, F=1 is chosen. To satisfy assumption A3, $\gamma_1(y) = 0.075$, $\gamma_2(y) = 0.075$ and $\rho_i(.)$ is set zero, too.

So, theorem 4.1 can be applied by mentioned data for the system of two inverted pendulum coupled by spring. Thus, matrix W must be constructed and related condition must be tested in the theorem.

$$W \cong \begin{bmatrix} 1.4885 & 0.2557 & -1.6440 & -0.2340 \\ 0.2557 & 1.7443 & -0.2340 & -1.6440 \\ -1.6440 & -0.2340 & 2.0000 & 0.0000 \\ -0.2340 & -1.6440 & 0.0000 & 2.0000 \end{bmatrix} \tag{60}$$

That the condition $W + W^T > 0$ is satisfied.

VII. Simulation Results

The MATLAB/Simulink model and programmes used for the simulation can be obtained by corresponding to the authors. The program of coupled-pendulums, as a sample, is presented in appendix. By changing the values of parameters in the "data file" and other *.m files, various simulation results can be obtained. The results obtained by applying control to the system using output feedback decentralized control are shown in figures 2-5. These figures present the response of the model to a pulse disturbance. Figures 2-3 present the values of angle of each pendulum and figures 4-5 present the values of angular velocities of each pendulum. Despite of existing the variation in values at the first steps, the system is stable. This is due to conditions used in the design are sufficient conditions.

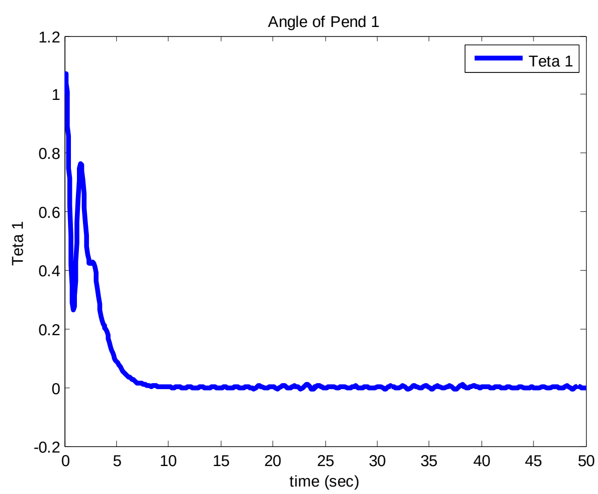

Fig 2. angle θ_1

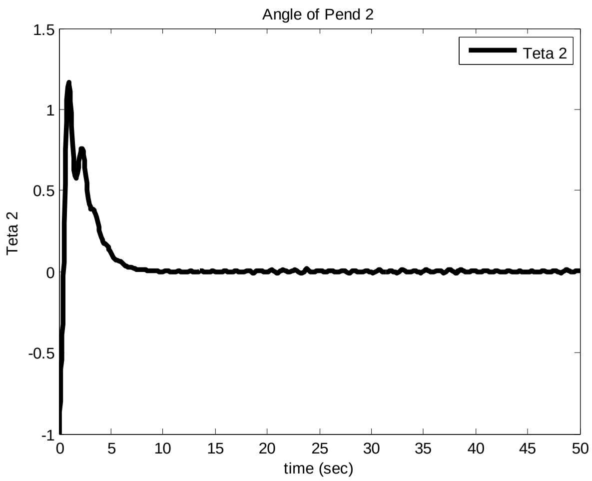

Fig 3. angle θ_2

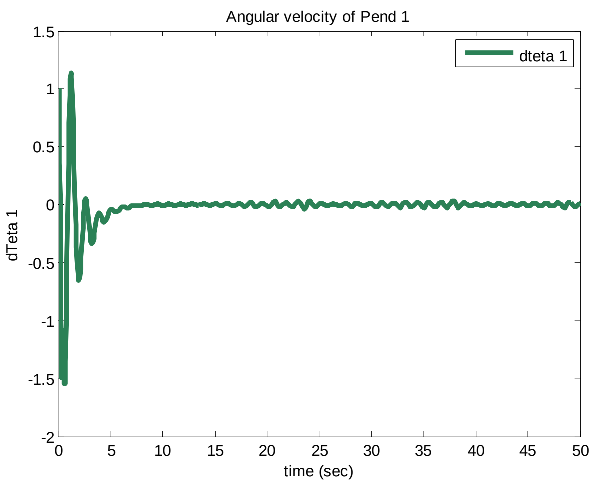

Fig 4. angular velocity of pendulum 1

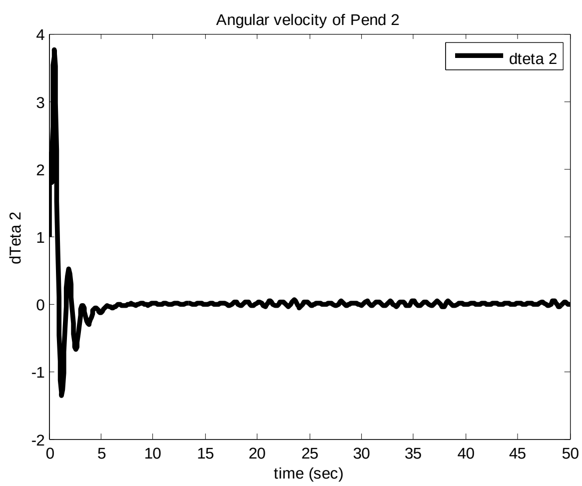

Fig 5. angular velocity of pendulum 2

VIII. Conclusion

We have presented a dynamic output feedback control scheme to stabilize a class of nonlinear interconnected systems. A system of two inverted pendulum coupled by spring is controlled using output feedback decentralized control.. As shown in figures, the variation of angle θ and angular velocity is stabilized. We mention that this method may be extended to the case where the dimensions of each subsystem are different by introducing a new similar structure. Last, but not the least, it is demonstrated that the system structure plays an important role in reducing the computation effect for the Lyapunov equation.

Appendix

```
function dx = coupled_pendulums(t,x)
dx = zeros(4,1);   % a column vector
l1=1;   % lenth
m1=1;   % mass

% pendulum 2
l2=0.8;  % lenth
m2=0.8 ; % mass

%L is the distance of two penduli
L=1.2;
kappa = 4.0;

%g is the gravity constant
g=9.81;
%%%%%%%%%%%%%%%%%%%%%%%%
ls=((L+l2*sin(x(3))-l1*sin(x(1)))^2+(l2*cos(x(3))-l1*cos(x(1)))^2)^0.5;
F=kappa*(ls-[L^2+(l2-l1)^2]^0.5);
phi=atan((-l2*cos(x(3))+l1*cos(x(1)))/(L+l2*sin(x(3))-l1*sin(x(1))));
K=[10.957  3.485  0   0
 0  0   7.500   2.375];
%K = [11.327  3.001  0   0 ;0  0   7.594   1.861];
u=-K*x+[.1;0];
dx(1) = x(2) ;
dx(2) = u(1)+(m1*g*(l1/2))*sin(x(1))+(l1)*F*cos(x(1)-phi);
dx(3) = x(4);
dx(4)= u(2)+(m2*g*(l2/2))*sin(x(3))-(l2)*F*cos(x(3)-phi);
```

Refrences

[1] Y. Guo, D. J. Hill and Y. Wang, "Nonlinear decentralized control of large-scale power systems". *Automatica* 36, pp1275-1289, 2000.
[2] T C Yang, J H Zhang and H yu, " A new decentralized controller design method with application to power system stabilizer design", *Control Engineering Practice*, Vol. 7, pp537-545, 1999.
[3] C. Peng, D. Yue, t-C Yang, " an example system for the study of advanced large-scale system control".
[4] Farzan Rashidi, " Decentralized Power System Stabilizer Design in Multimachine Power Networks Using Genetic Algorithm", *7th WSEAS International Conference on Automatic Control Modeling and Simulation*, March 2005.
[5] Lam, J. and Yang, G. H.," Balanced Model Reduction of symmetric Composite Systems", *International Journal of Control*, Vol.65, pp. 1031-1043, 1996.
[6] Qu, Z, and Dawson, D. M, " Robout Control of Cascaded and Individually Feedback Linearizable Nonlinear Systems", *Automatica*, Vol. 30, pp.1057-1046, 1994.
[7] Yan, X. G, Wang, J. J, Lu, X . an Zhang, S. Y.," decentralized Output Feedback Robust Stabilization for a Class of Nonlinear Interconnected Systems with Similarity", *IEEE Transactions on Automatic Control*, Vol. 43, pp. 294- 299, 1998.

[8] R. Sadeghian, P. Karimaghaee, A. R. Khayatian, " Controller Reduction of Discrete Linear Closed Loop Systems in a Certain Frequency Domain", *Proceeding of the 9th WSEAS Int. Conf. on Mathematical Methods and Computational Techniques in Electrical Engineering*, Arcachon, October 13-15, pp: 19-23, 2007.
[9] A. Zemliak, A. Michua, T. Markina, " Analysis of a Lyapunov Function Characteristics for Various Strategies of Designing Circuits", Recent Researches In Communications, Automation, Signal Processing, Nanotechnology, Astronomy and Nuclear Physics, available: www.wseas.org/search.
[10] A. Zemliak, E. Rios, "Stability Analysis of the Design Trajectories for the Generalized System Design Algorithm", *WSEAS Trans. On circuit on Systems*, Vol. 4, No. 2, 2005, pp.78-85
[11] X. G. Yan, J. Lam, H. S. Li, and I. M. Chen. "Decentralized Control of nonlinear Large-Scale Systems Using Output Feedback". *Journal of Optimization and Applications*: Vol, 104, No. 2, 459-475, February 2000.
[12] M.Ikeda and D. D. Siljak, "Optimality and robustness of linear quadratic control for nonlinear systems", *Automatica*, Vol. 26, no.3. pp. 499-511, 1990.

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