A Fuzzy PMCI Model for Productivity Improvement with a Survey in the Health Care Organization

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Abstract— In a competitive environment, health care organizations must continuously improve their productivity to sustain long-term growth and profitability. High productivity performance has been mostly assumed to be a natural outcome of successful health care management. The goods and services creation requires changes in the expended resources into the output goods and services. The efficiently of transforming input resources into goods and services depend on the productivity of the transformation process. However, it has been observed there is always vagueness or imprecision associated with the values of inputs and outputs. Therefore, it is difficult for a productivity measurement expert to specify the amount of resources and the outputs as exact scalar numbers. The present paper, applies fuzzy set theory to measure productivity of a hospital with PMCI method when numerical data cannot be specified in exact terms. The approach makes it possible to measure productivity of organizational units (including non-government and non-profit entities) when the expert inputs cannot be specified as exact scalar quantities.

Keywords— Fuzzy PMCI model; Vagueness; Productivity; Health care organizations.

I. Introduction

Any for-profit or non-profit organization requires a set of input resources in order to operate and survive. In return, it provides goods or value-adding services for its clients or stake-holders. The efficiency, with which it consumes the resources to provide those services, is measured by the productivity of the organization. The notion of productivity, therefore, focuses on exploring the relationship between the results achieved and the resources expended to achieve those results.

In its basic form, the productivity is measured by the ratio of outputs (often goods or services) to the input resources (such as labor, capital, management, materials, energy, etc.). Two most common measures of productivity are total measure and partial measures. Total measure includes all the input resources used in achieving the desired outputs, whereas the partial measures focuses on an incomplete list of input factors. If a partial measure focuses on one factor only (e.g. output per labor hour), it is referred to as the single factor productivity measure, whereas including more than one factor gives multi factor productivity.

Sometimes, the use of single factor productivity can be misleading when there is a tradeoff involved among multiple inputs. For example, an organization may procure a better and more expensive software or technology that requires fewer manual processing by its staff. Thus, it is possible to increase labor productivity but at the expense of increased technological costs. Therefore, if an improvement in the single factor productivity has been achieved, it is important to carefully examine the factors responsible for it or alternatively, have a more holistic approach towards productivity in terms of multi or total productivity measure.

The main reasons for process productivity measurement are to monitor and control the organizational performance, judge the effectiveness of our decisions and to create a metric that causes the behavioral change among the employees leading towards a productive unit. Measuring productivity is not an easy task, mainly because both output as well inputs are difficult to measure or count in a meaningful way. At first instance, the determination of the output seems quiet straightforward but due to problems in measuring the quality in the service sector and the prohibitive costs of surveys; it becomes difficult to specify the exact amount of satisfactory output.

Frequent service offerings, price and fees fluctuations, service aggregation are some of the other issues that further add to the problem. Personnel, capital and management are considered to be the critical inputs to enhance productivity. Inappropriate time standards, disparity in employee skills and motivation levels, flexibility in over and underutilization of budgets, technological changes, economies of scale, unaccounted hidden costs and the difficulties in measuring the efforts of management, all these factors make it increasingly more difficult to ascertain the systems inputs in precise numerical terms. Thus determining both the system inputs as well as the results achieved is an onerous task and it is highly unlikely that an expert would be able to specify them in precise quantities. The fundamental flaw in the traditional approaches is that the imprecision of parameters is ignored. If such imprecision has not been incorporated into the productivity measurement model, it may result in misrepresentation of a situation which further leads to erroneous results. A model that explicitly incorporates the effects of such vagueness may is appropriate under these conditions. Fuzzy set theory has proved to be a very valuable tool to handle this type of imprecision or vagueness in data.

II. LITERATURE REVIEW

Miller and Rao [1] analyzed profit-linked productivity models at the firm level. The issue of productivity measurement under multiple criteria has been explored in Ray and Sahu [2] and the sensitivity of productivity measurement in a multi-product setting has been discussed in Ray and Sahu [3]. Garrigosa and Tatje [4] performed the comparative study between profits and productivity as the two measures for performance. Sudit [5] discussed various productivity measures applicable in diverse settings. Chiou et al. [6] utilized quality function deployment in an approach that measures the productivity of technology in the product development process.

Agrell and West [7] critically examined a set of relevant properties that a productivity index must satisfy in order to assess the performance of a decision-making unit. Neely et al. [8] and Singh et al. [9] provided fairly detailed reviews of the previous research on productivity measurement.

Suwignjo et al. [10] made use of tools such as cognitive maps, cause-effect diagrams, tree diagrams as well as analytical hierarchical process to quantify the effects of performance factors. Odeck [11] analyzed the efficiency and productivity growth of vehicle inspection services using DEA piecewise linear function and Malmquist indices. Ylvinger [12] presented multi-input and multi-output generalized structural efficiency measures based on linear programming DEA models to estimate the relative performance of an industry. Hannula [13] mentioned the trade-off between validity and practicality of productivity measures, and presented a practical method expressing total productivity as a function of partial productivity ratios with acceptable validity at an organizational unit level. Raa [14] presented an approach to quantify the inconsistency in aggregating the firm productivities through allocative efficiency and excess marginal productivities. Chavas and Mechemache [15] investigated the measures for technical, efficiency, allocative efficiency and price efficiency which can be conveniently summed into an overall efficiency measure. Cooper et al. [16] provided a fairly comprehensive account of applications of data envelop analysis (DEA) in performance measures. Majority of these publications do not address the vagueness or imprecision in data.

There are quite a few publications that explore the imprecise nature of the input-output data in productivity and efficiency measures. Chen et al. [17] applied fuzzy pattern recognition clustering techniques to determine productivity characters and a business unit is diagnosed through these characters. Joro et al. [18] showed that the DEA formulation to identify efficient units is similar to the multi-objective linear programming model based on the reference point approach to generate efficient solutions. Triantis and Girod [19] proposed a three stage approach to measure the technical efficiency in a fuzzy parametric programming environment by expressing input and output variables in terms of their risk-free and impossible bounds. Girod and Triantis [20] illustrated the implementation of a fuzzy set-based methodology that can be used to accommodate the measurement inaccuracies using risk-free and impossible bounds to represent the extremes for fuzzy input and output.

Triantis and Eeckaut [21] used fuzzy pair wise dominance to measure the distance of a production plan from a frontier. Cooper et al. [22] provided imprecise data envelop analysis (IDEA) that permits a mixture of imprecise and exact data. Cooper et al. [23] further extended it for assurance region and cone-ratio concepts by placing bounds on variables rather than data values. The approach is applicable to bounded data and data sets satisfying ordinal relations and has been illustrated through an application to branch offices of a telecommunication company in Korea. Cooper et al. [24] removed a limitation of IDEA and assurance region IDEA which required access to actually attained maximum values in the data, by introducing a dummy variable for normalization of maximal values.

Despotis and Smirlis [25] developed an approach to transform a non-linear DEA model to a linear programming equivalent, on the basis of the original data set, by applying transformations only on the variables. Despotis and Smirlis [25] model allows post-DEA discriminating among the efficient units by endurance indices and is an alternative to Cooper et al. [22]. Zhu [26] reviewed and compared two different approaches dealing with imprecise DEA; one using scale transformations and the second using variable alterations through an efficiency analysis. Zhu [26] presented these two approaches as improvements over Cooper et al. [23]. Triantis [27] proposed a fuzzy DEA approach to compute fuzzy non-radial technical efficiency measures. Kao and Liu [28] provided a fuzzy DEA procedure by transforming it into a crisp DEA model using the -cut concept of fuzzy set theory and the resulting efficiency measures are provided in terms of fuzzy sets. Kao and Liu [29] applied a maximizing–minimizing set method for fuzzy efficiency ranking of 24 university libraries in Taiwan. Lertworasirikul [30] and Lertworasirikul et al. [31] proposed two main approaches; a possibility approach and a credibility approach to resolve the problem of ranking fuzzy sets in fuzzy DEA models.

León et al. [32] developed fuzzy versions of the classical BCC-DEA model by using ranking methods based on the comparison of α-cuts. Entani et al. [33] and Wang et al. [34] changed fuzzy input - output data into intervals using α-level sets and suggested two interval-DEA models. Dia [35] fuzzy-DEA model requires the decision maker to specify an aspiration level and a safety α- level in order to transform it into a crisp DEA model. Kao and Liu [36] transformed fuzzy input and output data into intervals by using α-level sets and fuzzy extension principle and built a family of crisp DEA models for the intervals.

Soleimani-damaneh et al. [37] addressed some computational and theoretical pitfalls of the fuzzy DEA models and provided a fuzzy DEA model to produce crisp efficiencies for DMUs with fuzzy input and output data. You et al. [38] presented a fuzzy multiple objective programming approaches to imprecise data envelopment analysis (IDEA) with an increased discriminating power than available from Cooper et al. [22]. Wang et al. [39] proposed two new fuzzy DEA models constructed from the perspective of fuzzy arithmetic and the models are applied to evaluate the performances of eight manufacturing enterprises in China. As evident from this literature survey, most of the existing approaches that deal with imprecise nature of data, present several variations of the DEA approach in a fuzzy environment. DEA based approaches are optimization approaches in the sense that they identify the best set of weights to identify the maximum achievable efficiency for an organizational unit, rather than identifying its true efficiency. Secondly, DEA-based approaches provide a relative measure of efficiency amongst a set of decision making units (DMU's). These approaches compare the DMU's input and output against a composite input and output.

If a particular DMU uses more inputs than the composite, it is termed as inefficient and vice-versa. As a potential drawback, if one DMU has substantially higher performance than others, most of the DMU's (except the one with exceptional performance) are likely to be termed as inefficient. Similarly, a MU with an exceptionally low performance may render other DMU's as efficient, not encase of their performances but due to the relative nature of the measurement. Furthermore, when new DMU's enter or leave the system (e.g. a new member joining or leaving a supply chain), efficiencies need to be re-evaluated. This establishes the need to have an approach that measures real productivity of a system in an absolute sense and in an environment involving imprecision and vagueness of data. This is one area where the present paper intends to contribute.

The next section deals with some basic concepts of fuzzy set theory that have been used to develop the proposed framework to model productivity. The subsequent action presents a fuzzy set theoretic model for multi factor productivity. The proposed model is illustrated through an application to 13 branches of a credit union. The computational experience and some important observations drawn from this experience are discussed. Finally, concluding remarks and some directions for further research are presented. Chen *et al present a* model Reduction for Discrete Interval Systems Using Genetic Algorithms [40]. Chong et al created a projection Based Method for Sparse Fuzzy System Generation [41]. Mazilescu presented a real Time Control System based on a Fuzzy Compiled Knowledge Base [42].V. Grisales et al exhibited a defuzzification scheme suitable for digital hardware implementation [43].

III. FUZZY CONCEPTS

Since its inception by Lofti Zadeh [44], fuzzy logic has revolutionized the business world with its ability to model the imprecise decision making situations. This section presents some basic concepts in fuzzy set methodology that have been utilized to develop the proposed model in this paper. For details of these concepts, the reader is referred to Kaufmann and Gupta [45] and Zimmermann [46].

A. Fuzzy set and membership function

A fuzzy set A in X is characterized by a membership function, $\mu_A(x)$ which associates with each element in X, a real number in the interval [0,1] with the value of $\mu_A(x)$ at x representing the "grade of membership" of x in A.

B. Interval mathematics for fuzzy numbers

Consider a situation where the value of a given input, $x \in \Re$, is uncertain or vague. In this case, it might be logical to express the input as an interval [3], thereby indicating that the input is known to exist between two real numbers a_1 and a_2, as shown in Figure 1. The uncertain value, x, belongs to a closed bounded interval $[a_1, a_2]$. We can then define an interval number, A, as the set of real numbers x such that $a_1 \le x \le a_2$, or

$$A = [a_1, a_2] = \{ x \mid a_1 \le x \le a_2, x \in \Re \}. \quad (1)$$

Given that we can express an uncertain input as an interval number, the operations on this input value are then governed by the interval arithmetic operations. The basic operations are outlined below:

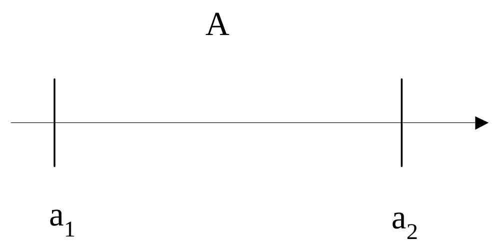

Figure 1: An interval number, A.

Addition of intervals

$$A + B = [a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2] \quad (2)$$

Subtraction of intervals

$$A - B = [a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1] \quad (3)$$

Multiplication of intervals

$$A \bullet B = [a_1, a_2] \bullet [b_1, b_2] = [\min(a_1b_1, a_1b_2, a_2b_1, a_2b_2), \max(a_1b_1, a_1b_2, a_2b_1, a_2b_2)] \quad (4)$$

Division of intervals

$$A \div B = [a_1, a_2] \div [b_1, b_2]$$

$$= [a_1, a_2] \bullet [\frac{1}{b_2}, \frac{1}{b_1}],$$
$$0 \notin [b_1, b_2] \quad (5)$$

Fuzzy numbers are a generalization of interval numbers [4]. We can interpret the exact value of x (expressed as an interval number, A) as being any number in the given interval, with all values equally possible. The generalization to a fuzzy number, $\tilde{A}$, would be that not all values in the interval are equally possible. The degree to which they are possible can then be interpreted as the membership function, i.e. the degree to which they are members of the interval. The membership function,

$$\mu_{\tilde{A}}: \mathbf{X} \rightarrow [0,1], \quad (6)$$

maps numbers in the interval to the interval of real numbers from 0 to 1, inclusive. There are many variations in describing fuzzy numbers. We shall confine our discussion to triangular fuzzy numbers. A triangular fuzzy number, $\tilde{A}$, is

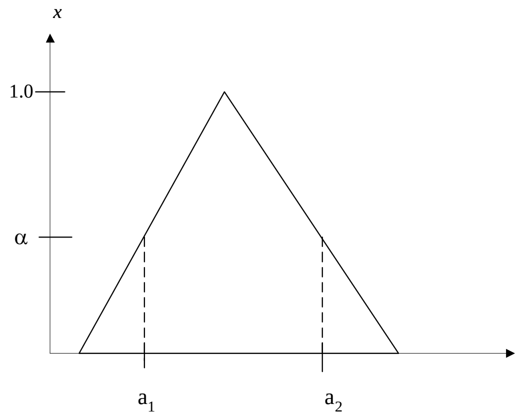

Figure 2: An α-cut of a fuzzy number.

Depicted in Figure 2. It is defined by the membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_M - a_1}, & \text{for } a_1 \le x \le a_M \text{, and} \\ \dfrac{x - a_2}{a_M - a_2}, & \text{for } a_M \le x \le a_2, \end{cases} \quad (7)$$

where $[a_1, a_2]$ is the supporting interval and the point $(a_M, 1)$ is the peak.

An α-cut of a fuzzy number $\tilde{A}$ is an interval number A_α that contains all the values of real numbers that have a membership grade in $\tilde{A}$ greater than or equal to the specified value of α. This can be written as

$$A_\alpha = [a_1, a_2]$$
$$= \{x \in \tilde{A} \mid \mu_{\tilde{A}}(x) \ge \alpha\}. \quad (8)$$

Thus, by taking an α-cut of a fuzzy number, one can process the operations on fuzzy numbers via the interval operations described in equations 1 through 4. It is interesting to note that the set of all α-cuts of any triangular fuzzy number is a family of nested intervals.

The level set of $\tilde{A}$ is the set of all levels $\alpha \in [0,1]$ that represent distinct α-cuts of the given fuzzy number $\tilde{A}$. Formally,

$$\Lambda_{\tilde{A}} = \{\alpha \mid \mu_{\tilde{A}}(x) = \alpha \text{ for some } x \in \tilde{A}\}, \quad (9)$$

Where $\Lambda_{\tilde{A}}$ denotes the level set of the fuzzy number $\tilde{A}$.

C. *Types of fuzzy systems*

These systems are a knowledge-based or rules-based system. The rule based is the heart of system that contain of rules. An if-then rule is an if-then diction which many its words are membership functions. The start point for made a fuzzy system is earning a set of fuzzy if-then rules earned by expert persons. The other stage is the combine of these rules. The fuzzy systems use different ways for combination of tem. Usually are used 3 kinds of systems:

Net fuzzy systems
TSK fuzzy systems
Justification and difuzzification systems

The main structure of net system shows below. The rule based shows the collection of fuzzy if-then rules. The fuzzy inference engine combines these rules of fuzzy set in internal environment to external environment in the base of fuzzy logical rules.

TSK systems uses easy math relation instead using the rules that in net systems use descriptive expression whit linguistic values. In fact, TSK system is the average weight from numbers of ports. For using net fuzzy system one any way is adding one justification at input and deffuzification at output. The result is showed below. This system covers the problems of net fuzzy system and TSK system. In this paper, aim is a system whit justification and difuzzification.

IV. PRODUCTIVITY

Productivity expresses the relationship between the output of goods and services (real output) and the various inputs required for production (e.g. labor and capital). Two important productivity indicators used are: labor productivity, that is, the ratio of real output to labor input, and capital productivity, the ratio of real output to stock of fixed capital used in the production process. However, these indicators are limited in the sense that they indicate the influence of only one factor of production at a time on productivity. An improvement over these partial indicators is the multifactor productivity which takes into account the simultaneous influences of several factors on production, including qualitative factors such as better management, improved quality of inputs and higher quality of goods.

Unit Labor Cost (ULC) is another important indicator of competitiveness which is defined as the remuneration of labor for producing one unit of real output. As ULC can also be expressed as the ratio of average compensation to labor productivity, it indicates how improvement in productivity offsets increases in average compensation.

1. Real output is given by value added at constant prices.

$$\text{Output index} = \frac{\text{Value added (constant price) in year n}}{\text{Value added in base year}} \times 100$$

2. Employment/Labor input

In the absence of total man hours, labor refers to the total number of persons engaged, that is employers, own account workers, contributing family workers and employees in any type of economic activity. Employment for year n is the average number of persons engaged in June of year (n) and June of year (n+1).

$$\text{Labor input index} = \frac{\text{Average number of persons engaged in year n}}{\text{Average number of persons engaged in base year}} * 100$$

3. Capital input

Capital refers to the net stock of investment in reproducible fixed assets. Reproducible fixed assets are investments in residential and non-residential building (excluding land), infrastructural work, machinery and equipment.

$$\textit{Capital input index} = \frac{\textit{Stock of fixed capital in year n}}{\textit{Stock of fixed capital in base year}} * 100$$

4. Labor Productivity

Labor productivity index shows the rate of change in output per person engaged.

$$\textit{Productivity Index} = \frac{\text{Output index}}{\text{Labor input index}} * 100$$

5. Capital productivity

The capital productivity index shows the rate of change in output per unit of capital.

$$\textit{Capital Productivity Index} = \frac{\text{Output index}}{\text{Capital input index}} * 100$$

6. Multifactor/Total factor productivity

Multifactor productivity (MFP)/Total factor productivity (TFP) index shows the rate of change in "productive efficiency", and is obtained as the ratio of the output to a weighted combination of labor and capital inputs. The limitation of partial productivity measures is that they attribute to one factor of production, changes in efficiency that are attributable to other factors. MFP reflects many influences including qualitative factors such as better management and improved quality of inputs through training and technology.

Multifactor productivity index (MPI) =

$$\frac{\text{Output index}}{\text{Multifactor input index}} * 100$$

$$A(t) = \frac{Q(t)}{\{WL(t) \times L(t)\} + \{WK(t) \times K(t)\}} * 100$$

Where

A (t) = Multifactor Productivity index in time t

Q (t) = Output index in time t

WL (t) = Labor's input share in time t (ratio of compensation of employees to value added)

L (t) = Labor input index in time t

WK (t) = 1- WL (t)

K (t) = Capital input index in time t

7. Unit Labor Cost

Unit labor cost is the remuneration of labor to produce one unit of output. It is computed as the ratio of the labor cost index to an index of production. The index shows the rate of change in labor cost per unit of output.

$$\textit{Unit Labor Cost Index} = \frac{\text{Labor Cost Index}}{\text{Output Index}} * 100 \text{ or}$$

$$\frac{\text{Average Compensation Index}}{\text{Labor Productivity Index}} * 100$$

For Competitiveness purposes, the exchange rate effect has to be taken into account. ULC is therefore computed both in local currency and in US dollar.

ULC index (US \$) = ULC index (MUR) / Exchange rate index of MUR/ US \$.

8. Hourly Labor Cost

Hourly labor cost is the ratio of compensation to total hours worked, inclusive of overtime. Compensation of employees comprises wages & salaries in cash and in kind, bonus, overtime and social contribution incurred by employers. The sources of data are Survey on Employment & Earnings carried out in March and for total hours worked, the September Survey of Employment, Earnings and Hours of work.

V. THE PMCI MODEL UNDER FUZZY ENVIRONMENT:

The hospital's managers intend to improve the productivity of this hospital by PMCI model. The following information have been obtained for our surveyed hospital

	Item	Average fuzzy number
1	Management	(49.31,76.81,94.63)
2	Sentry and reception	(37.92,50.14,75.79)
3	Expert medico	(84.02,91.81,98.14)
4	Nurse	(69.86,84.84,93.26)
5	Para clinical services	(59.87,70.27,81.67)
6	Services and hygiene	(35.46,44.97,56.30)
7	Official sector	(29.83,40.63,49.43)
8	Drugstore	(34.98,60.30,83.25)
9	Medical equipment	(59.81,78.85,89.74)
10	Sport space	(30.55,45.18,50.39)

	Item	extreme	middle
1	Management	12342	9566
2	Sentry and reception	9435	6354
3	Expert medico	12458	11653
4	Nurse	11689	10574
5	Para clinical services	10569	8754
6	Services and hygiene	6958	5245
7	Official sector	6452	4685
8	Drugstore	10542	7453
9	Medical equipment	11165	9564
10	Sport space	6098	6358

	Item	least	defuzzification
1	Management	6150	74.52
2	Sentry and reception	4789	53.36
3	Expert medico	10537	92.65
4	Nurse	8874	83.72
5	Para clinical services	7451	71.01
6	Services and hygiene	4450	43.34
7	Official sector	3779	40.01
8	Drugstore	4112	56.82
9	Medical equipment	7451	78.61
10	Sport space	3451	42.03

Table.1 data of the case study

The rules have been generated with the followings steps.

Step1. Output specification

Use There is provided a questionnaire and asks form sick persons that ask 3 question for each case

1) What is the measure of importance of item x? x_1

2) What is the extreme measure of importance of item x? x_2

3) What is the least measure of importance of item x? x_3

Which we have items in below

You consider the results as a (x_2 x_1 x_3) and then we obtain the result and introduce the important cone as important outputs

The number of pattern is 124 persons. The result is showed below: We grasp that the doctor, nurses, tools, management and service have most important that its 70%. So, we can select them for PMCI model. We define the particular for these outputs now.

Step2. Calculation of the outputs indexes

	Important output	index
1	Expert medico	$\frac{\text{The number of acquiescent sick}}{\text{total patients}}$
2	Nurse	The number of mistakes
3	Medical equipment	$\frac{\text{The number of devices}}{\text{the number of essential devices}}$
4	Management	$\frac{\text{The number of complaint at manager}}{\text{total complaint}}$
5	Para clinical services	The number of true specification total services

Table.2 indexes of outputs

It is defined important output particular for each of the important outputs in according to defined particular.

Step3. Rule Generation

For earning it, is used fuzzy system for each important outputs. We said the fuzzy system include rule based and inference engine, fuzzification, difuzzification.

In this paper, there is the contingency relationship for tools of medical. The rule based includes:

If efficiency is (65, 70, 75) then effectiveness is (-5, 0, 5)
If efficiency is (70, 75, 80) then effectiveness is (5, 10, 15)
If efficiency is (75, 80, 85) then effectiveness is (15, 20, 25)
If efficiency is (80, 85, 90) then effectiveness is (25, 30, 35)
If efficiency is (85, 90, 95) then effectiveness is (35, 40, 45)
If efficiency is (90, 95,100) then effectiveness is (45, 50, 55)
If efficiency is (60, 65, 70) then effectiveness is (-10,-5, 0)
If efficiency is (55, 60, 65) then effectiveness is (-15,-10,-5)
If efficiency is (50, 55, 60) then effectiveness is (-20,-15,-10)
If efficiency is (50, 55, 60) then effectiveness is (-25,-20,-15)

It's certain that effectiveness doesn't have direct relationship with efficiency. So sometimes these have inverse relationship.

Now, by inserting these data, efficiency numbers for different manners can is obtained. As you see, this picture is the medium graphic of on fuzzy in MATLAB. We can obtain the total efficiency the numbers for different effectiveness by this fuzzy system that the result is below:

Design the fuzzy systems for all important outputs are necessary.

The fourth stage: providing feedback reports.

As was said before, in this stage, firstly the information is collected in a specific period of time. Then in the base of the contingency relationship, the efficiency numbers is determined for each particular. Then total productivity is obtained by adding total productivity is obtained by adding total efficiency numbers of each output.

In according to calculated efficiency, the numbers which are needed for PMCI model are the related efficiency to medium and maximum effectiveness that is obtained them by using fuzzy systems from the contingency graph insert them to the productivity particular formula which will be point.

The whole productivity in the Hospital

For earning it, we must multiply related efficiencies which average effectiveness of one by one of particular in importance of it and add them together, the total should divides the whole productivity whit effectiveness maximum on its weight that is:

Wi =the measure of i in the productivity

εi=the earning efficiency from the necessity table for i in related to maximum effectiveness in a period of time.

N= the number of important factors in the productivity of whole of organization

For this case study:

$$p = \frac{\sum_{i=1}^{\mathrm{N}} \omega_{\mathrm{i}}\, \varepsilon_i}{\sum_{i=1}^{\mathrm{N}} \omega_i \mathrm{E}_i} = \frac{24.27*64+...+19.32*31.4}{24.27*65+...+19.32*44} =$$

$$\frac{4574.8}{5673.83} = 0.8063$$

VI. Conclusion

The present paper recognizes that measurements of system inputs and outputs for productivity measurement is a difficult task resulting in vagueness or imprecision in data. The paper proposes an approach based on fuzzy set theory to model this type of vagueness. The proposed approach provides a general model for productivity measurement. Because the measurement is necessary in improvement of productivity in each organization, so the way is important. PMCI technique is a model that has been used in several organizations, the fuzzy logic and fuzzy systems is used, because, they don't have certain outputs. In this paper, PMCI model at fuzzy environment is implied and can offer away that is useful in uncertain environment.

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