Overlapping Control System for Water Distribution Network

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***Abstract*—**In order to allow practical control of distribution in real time, this paper analyzes a sensitivity matrix that can be created using piecewise linearization and proposes a method for implementing decentralized distribution control using control systems decomposed from this sensitivity matrix. Implementing decentralized distribution control, the following two problems must be solved. The first problem is into what type of control ranges should be decomposed. The second problem is how to select control laws for each decentralized subsystem. In regard to the first problem, the present paper proposes an overlapping control structure based on a sensitivity matrix. Convergence of control is evaluated from a mathematical perspective for a centralized control structure, an overlapping control structure, and a separated control structure. The results show that convergence of control using an overlapping structure is superior to a separated structure. In regard to the second problem, adaptive control laws are derived based on an object characteristic model assuming a linear input-output relationship that includes unknown parameters.

***Keywords*—** Overlapping decomposition, Water distribution networks, Adaptive control, Convergence rate.

I. Introduction

THE principal duty of water service operation is the safe and stable supply of a daily necessity: water [1][2]. Concern about water quality safety by water suppliers has risen significantly in recent years coupled with a deterioration in quality of raw water and an increasing awareness of safety by the consumer. Thus, more sophisticated management of drinking water quality is required [3][4]. In addition, along with sufficient maintenance of urban infrastructure, the need for water distribution using appropriate pressure aimed at reducing leakage is strongly required. If water pressure in a distribution pipeline is too high, leakage from places such as pipe joints will occur easily. However, if the pressure is too low, the water supply may be cut off at the demand end due to insufficient pressure even when a faucet is open. Therefore, a problem exists in that it is necessary to maintain an appropriate pressure by opening and closing multiple valves at the operation end in the distribution pipe network in response to temporal variations in demand. This is distribution control. In distribution control, it is not sufficient to control single demand ends in the system. Instead, the water pressure in regions extending two-dimensionally (locally) is controlled.

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Thus, with the objective of leakage reduction in distribution systems, it is hoped that pressure optimization control will be realized. One method for implementing this control is to determine manipulated variables using nonlinear optimal calculation. However, since flow, pressure, and manipulated variables have mutually nonlinear relationships, optimal calculation of large-scale systems takes too much time and implementation in an online environment is difficult. From this perspective, the authors of the present paper earlier proposed a feedback control system using a piecewise linearization method [5]. This method makes it possible to scale down the amount of calculation by dealing with nonlinearity of water distribution networks having sensitivity coefficients based on demand levels. However, this method requires engineering to find the sensitivity coefficients again when the demand distribution changes. In addition, since this method stores multiple control gains based on demand size, file storage, management, and updating is also necessary. Adaptive control is a method of compensating for these drawbacks. However, since distribution systems are multi-input systems, it must be said that direct application of this method is still difficult.

In order to avoid this difficulty, it was proposed that a method of decentralized control be adopted, in which the system in problem is decomposed and an adaptive controller is placed in each subsystem. If this method is to be implemented, two problems must be answered. First, how should the pipes linking the network be decomposed? Second, what type of method should the adaptive controllers take? In the present paper, changes in measured variables in response to changes in manipulated variables were first examined as a sensitivity matrix, in order to answer the problem of decomposition of the system. Based on the results, a characteristic of the pipe network was found to be that the relationship between manipulated variables and measured variables simultaneously includes disjoint connection areas and tightly coupled relationships. Therefore, it was decided that an overlapping control structure would be constructed as a method that reflects this characteristic. Furthermore, the sensitivity matrix changes according to changes in demand, so that an adaptive control method that carries out pressure-fixation control while successively estimating the sensitivity matrix was proposed, convergence was evaluated mathematically, and control performance was evaluated using real pipe network data.

The following presents an outline of research related to the method proposed in the present paper that has been carried out up until the present time. Research exists that attempts to achieve adaptive control of water quality in a water distribution network [6][7]. This method aims to achieve water quality control using chlorine concentration based on the input-output relationship of disinfectant residual. In contrast, the research in the present paper relates to distribution control in which the measured pressure is controlled at the appropriate pressure through the operation of valves in response to pressure changes due to demand fluctuation, which differs from water quality control. In the present paper, overlapping decomposition of the pipe network in question is carried out, and adaptive control logic is adopted in each of the overlapping decomposed regions, so the control framework is also different from that used in the previous research.

In regard to previous research on overlapping control, decentralized PI control has been applied to Web tension control systems, and Sakamoto et al. improved this control by proposing an overlapping control system [8]. In addition, decentralized suboptimal control using overlapping decomposition in interconnected power systems has been proposed [9]. There is also research regarding the stability analysis of nonlinear dynamic systems showing that stability can be established using disjoint decomposition of the system and Lyapunov functions [10][11][12]. However, this previous research examined control objects that are relatively easy to decompose into subsystems and cannot be applied to the type of subject dealt with in the present paper, for which decomposition into subsystems is difficult. There is no previous research that proposes a method in which overlapping decomposition of a system linked in a nonlinear relationship by a pipe network is carried out and the subsystems are controlled using adaptive controllers, as shown in the present paper.

There is research that attempts to implement PID control and a dynamic programming approach in gas distribution control controlling gas volume and gas flow rate using valve operation [13][14].

In addition, a number of studies have examined adaptive control, such as application to nonlinear, large-scale systems [15][16][17], but there has been no research into overlapping control of distribution pipe networks, as proposed in the present paper.

In Section 2, an overlapping control structure based on a sensitivity matrix is proposed. Convergence of control is evaluated from a mathematical perspective for a centralized control structure, an overlapping control structure, and a separated control structure, and a separated control structure. In overlapping control, regions where measurement points are tightly coupled to manipulated variables have an information structure in which measurement information is overlapped and measured using multiple controllers. A method of analysis of mathematical convergence is shown, and the results show that convergence of control using an overlapping structure is superior to a separated structure.

Section 3 shows a pipe network adaptive control strategy for a control system installed in decentralized controllers based on a linearized adaptive control model that carries out control while adaptively and successively estimating control gains. In addition, when implementing overlapping control and separated control, it is essential to determine the range covered by each controller. An engineered approach that determines this range using numerals from a sensitivity matrix that indicates the influence of manipulated variables and measurements is shown.

In Section 4, Determining the structure of the overlapping decomposition means determining the range covered by each controller, which is determined based on the centralized control sensitivity matrix.

Section 5 summarizes the research results in conclusion.

II. Overlapping Decomposition Method

A. Formulation of distribution control problem

First, the mathematical model to which the pipe network physically conforms shall be described. There are two types of basic mathematical model for pipe networks: an equation that maintains flow balance at all nodes and the Hazen-Williams equation, which relates the flow rate to the nonlinear pressure drop.

The set of nodes that are supply points for the distribution network (distribution reservoirs, etc.) and the set of all other nodes are represented by N_{in} and N, respectively. The flow rate in pipe j is taken as x_j, the inflow (equivalent to the total amount of water supplied by the distribution reservoir) at node i is taken as w_i, and the outflow (demand) at node i is taken as y_i. In addition, the set of pipes starting at node i and the set of pipes ending at node i are represented by $A^+(i)$ and $A^-(i)$, respectively. On this occasion, the flow balance equation is represented as follows:

$$\sum_{j\in A^-(i)} x_j - \sum_{j\in A^+(i)} x_j = \begin{cases} -w_i & (i \in N_{\text{in}}) \\ y_i & (i \in N) \end{cases} \tag{1}$$

The unit for the flow rate in the following equation is m^3/s. Pressure refers to the pressure head, and the unit for the pressure head is m. If the set of pipes is taken as B, the pressure at node i is taken as p_i, the start and end points of pipe j are taken as $s(j)$ and $e^-(j)$, respectively, and the resistance of pipe j is taken as R_j, then the pressure balance equation is as follows:

$$P_{s(j)} - P_{e(j)} = R_j \left| x_j \right|^{\alpha-1} \cdot x_j \quad (j \in B) \tag{2}$$

Using the Hazen-Williams equation, the pipe resistance R_j is obtained as follows:

$$R_j = 10.666 C_j^{-1.85} D_j^{-4.87} L_j \tag{3}$$

$$\alpha = 1.85 \tag{4}$$

where C_j, D_j, and L_j represent, respectively, the coefficient of velocity, the diameter, and the length of pipe j. If valves or pumps are installed in the pipe, then a second term representing the pressure fall/rise due to valves or pumps is added to the right-hand side of Equation (2) above.

Pipe network analysis using minimum cost flow calculus is used in the analysis of steady flow in a water distribution network [18].

In an attempt to realize the abovementioned general mathematical model for pipe networks, a distribution control model is constructed from the viewpoint of conducting distribution control using controllers, as follows. Temporally fluctuating demand is rewritten as $\mathbf{u}_D(k) \in R^m$ by inserting time variable *k*. In addition, due to temporally fluctuating demand, a pressure change occurs in the pipe network. The objective of distribution control is to fix this pressure change using controllers. The variable that the controller can operate in order to fix the pressure is the resistance coefficient of valves (manipulated variable) $\mathbf{u}_V(k) \in R^l$. When described within the range required to develop the discussion in the present paper, the physical properties of the distribution pipe network can be expressed as a nonlinear pipe network state equation, as follows:

$$\mathbf{f}(\mathbf{x}_C(k),\ \ \mathbf{u}_V(k),\ \ \mathbf{u}_D(k)) = \mathbf{0} \tag{5}$$

where $\mathbf{x}_C(k) \in R^l$ is a co-tree flow, l is the number of co-tree pipes, $\mathbf{u}_V(k) \in R^l$ is the resistance coefficient of the valves (manipulated variable), n is the number of valves, $\mathbf{u}_D(k) \in R^m$ is the demand, m is the number of demand ends, and k is the time.

The vector dimension of this function $\mathbf{f}$ is l, which means that function $\mathbf{f}$ consists of l nonlinear algebraic equations.

Information obtained from pressure gauges in the pipe network can be expressed as a measurement system observation equation, as follows:

$$\mathbf{z}(k) = \mathbf{h}(x_C(k),\ \ \mathbf{u}_V(k),\ \ \mathbf{u}_D(k)) \tag{6}$$

where $\mathbf{z}(k) \in R^s$ is the measurement information. The vector dimension of Equation (6) is *s*, which means that *s* items of information are measured simultaneously.

The general pipe network equations, Equations (1) through (4), can be rewritten as Equations (5) and (6) from the perspective of distribution control. In other words, demand, which forms a disturbance in the control system, and a manipulated variable that can be operated by a controller are represented explicitly, and the physical properties of the pipe network are summarized as a state equation and an observation equation. However, note that Equations (5) and (6) are still nonlinear.

Because the objective of distribution control is pressure fixation, the following mathematical control index is taken:

$$J = (\mathbf{z}_0 - \mathbf{z}(k))^T(\mathbf{z}_0 - \mathbf{z}(k)) \tag{7}$$

where $\mathbf{z}^0 \in R^s$ is the control target value for pressure.

Thus, the manipulated variable that minimizes the control target equation (7) is determined from the pipe network equation (5) and the observation equation (6).

In regard to the control problem, linear control laws are presented next. When deriving control laws for online control, it is permissible to take the demand as constant, taking into account the slowness of change of state of the pipe network due to changes in demand compared to the control cycle. If demand $\mathbf{u}_D$ is assumed to be constant in the pipe network equation (5), then the flow $\mathbf{x}_C$ becomes a function of manipulated variable $\mathbf{u}_V$, and so observation equation (6) also becomes a function of $\mathbf{u}_V$. Therefore, Equation (6) may be written as

$$\mathbf{z}(k) = H(\mathbf{u}_V(k)) \tag{8}$$

If this is linearized by $\mathbf{u}_V(k)$, we have:

$$\Delta\mathbf{z}(k) = \frac{\partial H}{\partial \mathbf{u}_V}(\mathbf{u}_V(k))\Delta\mathbf{u}_V(k) \tag{9}$$

If the above equation is used to find the manipulated variable that minimizes the control target of equation (7):

$$\begin{aligned} &\mathbf{u}(k+1) = \mathbf{u}(k) + \frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))^+(\mathbf{z}_0 - H(\mathbf{u}(k))) \\ &\frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))^+ = -\left(\frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))^T \frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))\right)^{-1} \frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k)) \end{aligned} \tag{10}$$

where k is the calculation step, and each time instant of the system is taken as one calculation step.

Equation (10) is a linear control strategy derived under the assumption of constant demand. In reality, since the coefficient in Equation (10) also changes along with changes in demand, it is preferable to estimate this coefficient while determining the manipulated variable. However, since water distribution systems are multi-input, multi-output systems, the amount of calculation becomes excessive if adaptive control is applied directly without change. For this reason, a method for a type of decentralized control in which the system in problem is decomposed to an extent that adaptive control calculations can be executed using a microcomputer and adaptive control is applied to each decomposed subsystem will be considered. In implementing decentralized control, the problem of how to decompose the system arises. Therefore, a basic study of decomposition methods focusing on differences in information structure shall be carried out.

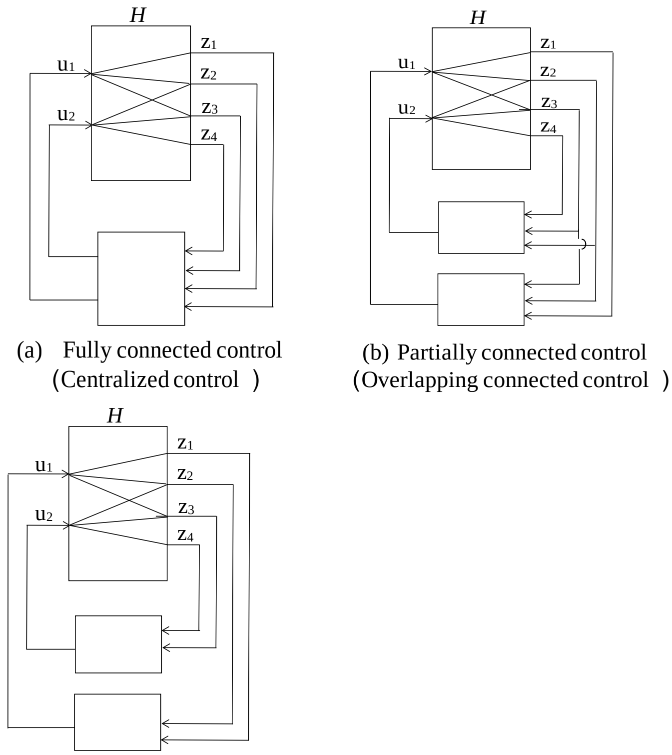
B. Basic study of decomposition

Generally, when considering system decomposition, the multiple subsystems obtained by decomposition do not intersect. In other words, full decomposition, in which subsystems do not overlap each other, has often been studied. However, in the decomposition of real systems, it may sometimes be more natural to carry out overlapping decomposition due to the physical properties of the system. Here, due to differences in information structure, overlapping control and separated control are established as decomposition methods, and a comparative evaluation of these methods is conducted for convergence while matching the case of centralized control.

As a basic examination, a distribution system with a simple structure, as shown in Fig.1, is considered:

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \mathrm{H}(u_1, u_2) = \begin{bmatrix} h_1(u_1) \\ h_2(u_1, u_2) \\ h_3(u_1, u_2) \\ h_4(u_2) \end{bmatrix} \tag{11}$$

The input-output structure of the control object H illustrated in Fig.1 shows that there is not a mutually disjoint connection. In other words, in the pipe network of interest, the measured variables z have a strong, nonlinear relationship with the manipulated variables u, as shown in Equations (2), (3), and (4), and are not mutually disjoint connections. On the other hand, observation point z_1, for example, is not influenced by manipulated variable u_2, which shows that some disjoint connection areas are inherent in the structure. Thus, the characteristics of the pipe network structure are represented by Equation (11) in preparation for the mathematical analysis that will be discussed later.



(a) Fully connected control (Centralized control)

(b) Partially connected control (Overlapping connected control)

(c) Separated control

Fig.1 Scheme of Control

Taking the control target value as z_0, the linear control strategy can generally be expressed as follows:

$$\mathbf{u}(k+1) = \mathbf{u}(k) + Q(\mathbf{u}(k))(\mathbf{z}_0 - H(\mathbf{u}(k))) \tag{12}$$

$Q(\mathbf{u}(k))$: First derivative at $\mathbf{u}(k)$

First, if control is carried out using one controller, as in the centralized control in Fig.1(a), the control law is the same as Equation (10).

Next, in Fig.1(b), the structure is such that each controller performs control by receiving information from observation points that are influenced by a manipulated variable controlled by the respective controller, and the measurement information is overlapped in this control (referred to as overlapping control). The control laws for each controller can be written as follows:

$$\begin{pmatrix} \frac{\partial h_1}{\partial u_1} \\ \frac{\partial h_2}{\partial u_1} \\ \frac{\partial h_3}{\partial u_1} \end{pmatrix} (u_1(k+1) - u_1(k)) = \begin{bmatrix} z_{01} - h_1(u_1(k)) \\ z_{02} - h_2(\mathbf{u}(k)) \\ z_{03} - h_3(\mathbf{u}(k)) \end{bmatrix} \tag{13}$$

$$\begin{pmatrix} \frac{\partial h_2}{\partial u_2} \\ \frac{\partial h_3}{\partial u_2} \\ \frac{\partial h_4}{\partial u_2} \end{pmatrix} (u_2(k+1) - u_2(k)) = \begin{bmatrix} z_{02} - h_2(\mathbf{u}(k)) \\ z_{03} - h_3(\mathbf{u}(k)) \\ z_{04} - h_4(u_2(k)) \end{bmatrix} \tag{14}$$

These laws can be combined and expressed by the following equation:

$$\begin{aligned} &\mathbf{u}(k+1) - \mathbf{u}(k) \\ &= -\begin{pmatrix} \sum_{i=1}^{3} \frac{\partial h_i^2}{\partial u_1} & 0 \\ 0 & \sum_{i=2}^{4} \frac{\partial h_i^2}{\partial u_2} \end{pmatrix}^{-1} \frac{\partial H^T}{\partial \mathbf{u}} (\mathbf{z}_0 - H(\mathbf{u}(k))) \\ &= -\left(\frac{\partial H^T}{\partial \mathbf{u}} \frac{\partial H}{\partial \mathbf{u}} - \begin{pmatrix} 0 & \sum_{i=2}^{3} \frac{\partial h_i}{\partial u_1} \frac{\partial h_i}{\partial u_2} \\ \sum_{i=2}^{3} \frac{\partial h_i}{\partial u_1} \frac{\partial h_i}{\partial u_2} & 0 \end{pmatrix} \right)^{-1} \frac{\partial H^T}{\partial \mathbf{u}} (\mathbf{z}_0 - H(\mathbf{u}(k))) \end{aligned} \tag{15}$$

Here,

$$\frac{\partial H^T}{\partial \mathbf{u}} = \begin{pmatrix} \frac{\partial h_1}{\partial u_1} & \frac{\partial h_2}{\partial u_1} & \frac{\partial h_3}{\partial u_1} & 0 \\ 0 & \frac{\partial h_2}{\partial u_2} & \frac{\partial h_3}{\partial u_2} & \frac{\partial h_4}{\partial u_2} \end{pmatrix} \tag{16}$$

Next, respective control laws for separated control, as shown in Fig.1(c), can be expressed as follows:

$$\begin{bmatrix} \frac{\partial h_1}{\partial u_1} \\ \frac{\partial h_2}{\partial u_1} \end{bmatrix} (u_1(k+1) - u_1(k)) = \begin{bmatrix} z_{01} - h_1(u_1(k)) \\ z_{02} - h_2(\mathbf{u}(k)) \end{bmatrix} \tag{17}$$

$$\begin{bmatrix} \frac{\partial h_3}{\partial u_2} \\ \frac{\partial h_4}{\partial u_2} \end{bmatrix} (u_2(k+1) - u_2(k)) = \begin{bmatrix} z_{03} - h_3(\mathbf{u}(k)) \\ z_{04} - h_4(u_2(k)) \end{bmatrix} \tag{18}$$

These equations can be combined and rewritten as follows:

$$\mathbf{u}(k+1)-\mathbf{u}(k)$$
$$=-\begin{pmatrix}\sum_{i=1}^{2}\frac{\partial h_i^2}{\partial u_1} & 0\\ 0 & \sum_{i=3}^{4}\frac{\partial h_1^2}{\partial u_2}\end{pmatrix}^{-1}\begin{pmatrix}\frac{\partial h_1}{\partial u_1} & \frac{\partial h_2}{\partial u_1} & 0 & 0\\ 0 & 0 & \frac{\partial h_3}{\partial u_2} & \frac{\partial h_4}{\partial u_2}\end{pmatrix}(\mathbf{z}_0-H(\mathbf{u}(k)))$$
$$=-\left(\frac{\partial H^T}{\partial H}\frac{\partial H}{\partial \mathbf{u}}-\begin{pmatrix}\frac{\partial h_3^2}{\partial u_1} & \sum_{i=2}^{3}\frac{\partial h_i}{\partial u_1}\frac{\partial h_i}{\partial u_2}\\ \sum_{i=2}^{3}\frac{\partial h_i}{\partial u_1}\frac{\partial h_i}{\partial u_2} & \frac{\partial h_2^2}{\partial u_2}\end{pmatrix}\right)^{-1}\left(\frac{\partial H}{\partial \mathbf{u}}-\begin{pmatrix}0 & 0 & \frac{\partial h_3}{\partial u_1} & 0\\ 0 & \frac{\partial h_2}{\partial u_2} & 0 & 0\end{pmatrix}\right)$$
$$+(\mathbf{z}_0-H(\mathbf{u}(k))) \tag{19}$$

A basic study of control convergence for the above three control structures shall be carried out. Here, it is assumed that

$$\frac{\partial h_2}{\partial u_i},\frac{\partial h_3}{\partial u_i}\sim 0(\varepsilon) \qquad i=1,2 \tag{20}$$

This means that the manipulated variables u_1 and u_2 are loosely coupled to control variables z_1 and z_2. The linear control strategy equation (12) is rewritten as an equation that represents convergence towards the optimum point $\mathbf{z}_0 = H(\mathbf{u}^*)$.

$$\mathbf{u}(k+1)-\mathbf{u}^*$$
$$=\mathbf{u}(k)-\mathbf{u}^*+Q(\mathbf{u}(k))(\mathbf{z}_0-H(\mathbf{u}(k)))-Q(\mathbf{u}(k))(\mathbf{z}_0-H(\mathbf{u}^*))$$
$$=\mathbf{u}(k)-\mathbf{u}^*+Q(\mathbf{u}(k))(H(\mathbf{u}^*)-H(\mathbf{u}(k)))$$
$$=\mathbf{u}(k)-\mathbf{u}^*+Q(\mathbf{u}(k))\frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))(\mathbf{u}^*-\mathbf{u}(k))+Q(\mathbf{u}(k))S(\xi)(\mathbf{u}^*-\mathbf{u}(k))(\mathbf{u}^*-\mathbf{u}(k)) \tag{21}$$

where $S(\xi)$ is taken as the second derivative of $H(\mathbf{u})$ with respect to ξ between $\mathbf{u}^*$ and $\mathbf{u}(k)$.

Using this equation, first, in order to examine the convergence of centralized control:

$$Q(\mathbf{u}(k))=\frac{\partial H}{\partial \mathbf{u}}(\mathbf{u}(k))^+ \cdot$$

Thus, Equation (21) becomes as follows:

$$\mathbf{u}(k+1)-\mathbf{u}^* = Q(\mathbf{u}(k))S(\xi)(\mathbf{u}^*-\mathbf{u}(k))(\mathbf{u}^*-\mathbf{u}(k)) \tag{22}$$

Taking this absolute value, convergence in the neighborhood of the optimum point can be written as follows:

$$|\mathbf{u}(k+1)-\mathbf{u}^*| \leqq |C||\mathbf{u}^*-\mathbf{u}(k)|^2 \tag{23}$$

Here, $|C|$ is taken to be $|Q(u(k))S(\xi)|<|C|$.

Therefore, in centralized control, the manipulated variable converges to $\mathbf{u}^*$ by the speed of the square.

In overlapping control, from Equation (15), we obtain

$$Q(\mathbf{u}(k))$$
$$=-(A-\delta A_P(\varepsilon^2))A^{-1}\frac{\partial H^T}{\partial \mathbf{u}} \tag{24}$$
$$=-(A^{-1}+A^{-1}\delta A_P(\varepsilon^2)A^{-1})\frac{\partial H}{\partial \mathbf{u}}-O(\varepsilon^4)$$

Here, convergence can be examined using Equation (21) by writing the following:

$$A=\frac{\partial H^T}{\partial \mathbf{u}}\frac{\partial H}{\partial \mathbf{u}},$$
$$\mathbf{u}(k+1)-\mathbf{u}^*$$
$$=-\left\{A^{-1}\delta A_p(\varepsilon^2)+O(\varepsilon^4)\frac{\partial H}{\partial u}\right\}(\mathbf{u}^*-\mathbf{u}(k))+C'(\mathbf{u}^*-\mathbf{u}(k))^2 \tag{25}$$

Here, C' is taken as the product of $Q(\mathbf{u}(k))$ in Equation (21) and the second derivative of $H(\mathbf{u})$.

Taking the absolute value, convergence of overlapping control is as follows:

$$|\mathbf{u}(k+1)-\mathbf{u}^*|$$
$$\leqq \left\{\left|A^{-1}\delta A_P(\varepsilon^2)\right|+O(\varepsilon^4)\frac{\partial H}{\partial u}\right\}|\mathbf{u}^*-\mathbf{u}(k)|+|C'||\mathbf{u}^*-\mathbf{u}(k)|^2 \tag{26}$$

Furthermore, from Equation (19), separated control can be written as follows:

$$Q(\mathbf{u}(k))$$
$$=-(A-\delta A_s(\varepsilon^2)^{-1}\left(\frac{\partial H^T}{\partial \mathbf{u}}-\delta B(\varepsilon)\right)$$
$$=-\left(A^{-1}\frac{\partial H^T}{\partial \mathbf{u}}-A^{-1}\delta A_s(\varepsilon^2)A^{-1}\frac{\partial H^T}{\partial \mathbf{u}}-A^{-1}\delta B(\varepsilon)+O(\varepsilon^3)\right) \tag{27}$$

Here,

$$\delta A_s(\varepsilon^2)=\begin{pmatrix}\frac{\partial h_3^2}{\partial u_1} & \sum_{i=2}^{3}\frac{\partial h_i}{\partial u_1}\frac{\partial h_i}{\partial u_2}\\ \sum_{i=2}^{3}\frac{\partial h_i}{\partial u_1}\frac{\partial h_i}{\partial u_2} & \frac{\partial h_2^2}{\partial u_2}\end{pmatrix},$$
$$\delta B(\varepsilon) = \begin{pmatrix}0 & 0 & \frac{\partial h_3}{\partial u_1} & 0\\ 0 & \frac{\partial h_2}{\partial u_2} & 0 & 0\end{pmatrix}$$

Substituting the above expression into Equation (17), we obtain:

$$|\mathbf{u}(k+1)-\mathbf{u}^*|$$
$$\leqq \left\{\left|A^{-1}\delta A_s(\varepsilon^2)\right|+\left|A^{-1}\delta B(\varepsilon)\frac{\partial H^T}{\partial \mathbf{u}}\right|+O(\varepsilon^3)\frac{\partial H}{\partial \mathbf{u}}\right\}|\mathbf{u}(k)-\mathbf{u}^*|+|C''||\mathbf{u}^*-\mathbf{u}(k)|^2 \tag{28}$$

Here, C'' is taken as the product of $Q(u(k))$ in Equation (23) and the second derivative of $H(u)$.

Comparing the convergence of Equations (23), (26), and (28), Equation (23) does not contain the first-order term $|\mathbf{u}^* - \mathbf{u}(k)|$, and the order of the coefficient of $|\mathbf{u}(k) - \mathbf{u}^*|$ is lower in Equation (26) than in Equation (28). Therefore, convergence can be evaluated as follows:

$$\text{Centralized} \succ \text{Overlapping} \succ \text{Separated} \tag{29}$$

III. Strategy for pipe network adaptive control

A. Control model for an adaptive controller

It was shown in the previous section that overlapping control, in which controllers are placed in each overlapping decomposed subsystem, is effective in terms of convergence for controllers that do not have the computing power to be able to cover the entire system. However, this is the case when the demand is constant and the sensitivity coefficient for this demand is known. In real problems, the sensitivity coefficient gradually changes along with changes in demand. Therefore, we describe the control model that each controller should possess in this case:

$$S_i : \Delta \mathbf{z}_i(t+1) = H_i \Delta \mathbf{u}_i(t) + \mathbf{c}_i \tag{30}$$

where $\mathbf{c}_i$ is an interference term between subsystems, summarized as variables, that are not explained by the manipulated variables covered by the controller in problem, when information for manipulated variables covered by other controllers cannot be obtained. As the control method in each controller, the sensitivity coefficient and the interference term are estimated from manipulated variable information and measurement information that is available to the controller in problem, and control is performed on this basis.

Next, an adaptive control strategy is derived using the self-tuning regulator (STR) method in which control is performed by estimating process parameters $H_i, \mathbf{c}_i$ in Equation (30) and determining the control parameters.

B. Pipe network adaptive control strategy

First, we show a process identification part for estimating, from process input-output, the sensitivity coefficient Hi and $\mathbf{c}i$, which are unknown parameters among the parameters comprising the adaptive control law. In Equation (30), if $t = 1, 2, \cdots, k-1$, the following equation is obtained:

$$\begin{bmatrix} \Delta \tilde{z}_i(1) \\ \Delta \tilde{z}_i(2) \\ \vdots \\ \Delta \tilde{z}_i(k) \end{bmatrix} = \begin{bmatrix} 1 & \Delta \mathbf{u}_i^T(0) \\ 1 & \Delta \mathbf{u}_i^T(1) \\ \vdots & \vdots \\ 1 & \Delta \mathbf{u}_i^T(k+1) \end{bmatrix} \begin{bmatrix} \mathbf{c}_i^T \\ H_i^T \end{bmatrix} + \begin{bmatrix} \varepsilon_i^T(1) \\ \varepsilon_i^T(2) \\ \vdots \\ \varepsilon_i^T(k) \end{bmatrix} \tag{31}$$

$$\tilde{Z}_i(k) = Ui(k)\Theta_i^T + E_i(k) \tag{32}$$

where $\varepsilon_i(t)$ is the measurement noise at time t, $\Delta \tilde{\mathbf{z}}(t)$ is the measurement at time t, $\hat{\Theta}_i(k)$ is the estimated value for Θ_i at time k, is determined so as to minimize the difference between $\hat{Z}_i(k) = U_i(k)\hat{\Theta}_i^T(k)$ and $\tilde{Z}_i(k)$. Specifically, the evaluation function of the square error is set as follows:

$$J_i(k) = \sum_{j=1}^{k} \lambda_i^{k-j} (\Delta \tilde{\mathbf{z}}_i(j) - \Delta \hat{\mathbf{z}}_i(j))^T (\Delta \tilde{\mathbf{z}}_i(j) - \Delta \hat{\mathbf{z}}_i(j)) \tag{33}$$

Here, the value to be estimated, Θ_i, changes slowly with time, so that a weighting $1 \geqq \lambda_i > 0$ was introduced. Applying the method of least squares to the above equation, if a sequential form is taken for real-time estimation, the following estimation equation can be derived:

$$\hat{\Theta}_i(k+1) = \hat{\Theta}_i(k) + \frac{1}{\Delta_i(k)} \left\{ \Delta \tilde{\mathbf{z}}_i(k+1) - \hat{\Theta}_i(k) \begin{bmatrix} 1 \\ \Delta u_i(k) \end{bmatrix} \right\} \left(\frac{F_i(k)}{\lambda_i} \begin{bmatrix} 1 \\ \Delta \mathbf{u}_i(k) \end{bmatrix} \right) \tag{34}$$

$$F_i(k+1) = F_i(k) - \frac{1}{\Delta_i(k)} \left(\frac{F_i(k)}{\lambda_i} \begin{bmatrix} 1 \\ \Delta \mathbf{u}_i(k) \end{bmatrix} \right) \left(\frac{F_i(k)}{\lambda_i} \begin{bmatrix} 1 \\ \Delta \mathbf{u}_i(k) \end{bmatrix} \right)^T \tag{35}$$

$$\Delta_i(k) = 1 + \begin{bmatrix} 1 \\ \Delta \mathbf{u}_i(k) \end{bmatrix}^T \frac{F_i(k)}{\lambda_i} \begin{bmatrix} 1 \\ \Delta \mathbf{u}_i(k) \end{bmatrix} \tag{36}$$

Where $\hat{\Theta}_i(k+1) = \left[\hat{\mathbf{c}}_i(k+1) \hat{H}_i(k+1) \right]$ (37)

Next, we show the control law for the controller unit. The control index of divided subsystem *i* is set as follows:

$$\begin{aligned} \text{Min.}\, I_i = {} & (\tilde{\mathbf{z}}_i(k+1) - \mathbf{z}_{0i})^T P_i (\tilde{\mathbf{z}}_i(k+1) - \mathbf{z}_{0i}) \\ & + (\mathbf{u}_i(k+1) - \mathbf{u}_i(k))^T W_i (\mathbf{u}_i(k+1) - \mathbf{u}_i(k)) \end{aligned} \tag{38}$$

where P_i and W_i are positive definite.

Using the above-mentioned estimated value, the subsystem control model is as follows:

$$\Delta \mathbf{z}_i(k+1) = \hat{H}_i(k+1) \Delta \mathbf{u}_i(k+1) + \hat{\mathbf{c}}_i(k+1) \tag{39}$$

Next, $\Delta \mathbf{u}_i(k+1)$, which minimizes control index, I_i, is found. The decrease in Equation (38), ΔI_i, can be written as follows:

$$\begin{aligned} \Delta I_i = {} & -2(\tilde{\mathbf{z}}_i(k+1) - \mathbf{z}_{0i})^T P_i \Delta \mathbf{z}_i(k+1) \\ & - \Delta \mathbf{z}_i^T(k+1) P_i \Delta \mathbf{z}_i(k+1) - 3\Delta \mathbf{u}_i^T(k+1) Q_i \Delta \mathbf{u}_i(k+1) \end{aligned} \tag{40}$$

In addition, if Equation (39) is substituted into Equation (40) and partial differentiation with respect to $\Delta \mathbf{u}_i(k+1)$ is carried out, setting $\frac{\partial \Delta I_i}{\partial \Delta \mathbf{u}_i(k+1)} = 0$, it is possible to find $\Delta \mathbf{u}_i(k+1)$, which minimizes I_i. In other words,

$$-(\tilde{\mathbf{z}}_i(k+1)-\mathbf{z}_{0i})\hat{H}_i^T(k+1)P_i-\hat{H}_i^T(k+1)P_i\hat{H}_i(k+1)\Delta\,\mathbf{u}_i(k+1)$$
$$-\hat{H}_i^T(k+1)P_i\hat{\mathbf{c}}_i(k+1)-3W_i\Delta\,\mathbf{u}_i^T(k+1) \;=\; \mathbf{0} \qquad (41)$$

$$\Delta\,\mathbf{u}_i(k+1)=$$
$$-(\hat{H}_i^T(k+1)P_i\hat{H}_i(k+1)+3W_i)^{-1}\hat{H}_i^T(k+1)P_i^T(\tilde{z}_i(k+1)-\mathbf{z}_{0i}+\hat{\mathbf{c}}_i(k+1)) \qquad (42)$$

$$\mathbf{u}_i(k+1)=\hat{G}(k+1)(\mathbf{z}_{0i}-\tilde{\mathbf{z}}_i(k+1)-\hat{\mathbf{c}}_i(k+1))+\mathbf{u}_i(k) \qquad (43)$$

where $\hat{G}(k+1)=(\hat{H}_i^T(k+1)P_i\hat{H}_i(k+1)+3W_i)^{-1}\hat{H}_i^T(k+1)P_i^T$ (44)

Equation (43) is the control law of the controller unit, and $\hat{G}_i(k+1)$ is the control gain.

Above is the control strategy for an adaptive controller placed in a decomposed subsystem. Next, this method is applied to a large-scale pipe network and verified through simulation.

IV. Decomposition Method in Real Pipe Network

Before verifying the method using the distribution system of City A as the subject, we will show specifically how overlapping decomposition will be carried out. The pipe network in problem has six distribution bases, eight valve installation points, 35 pressure observation points, 142 nodes, and 205 pipes, and is laid out as shown in Fig.2.

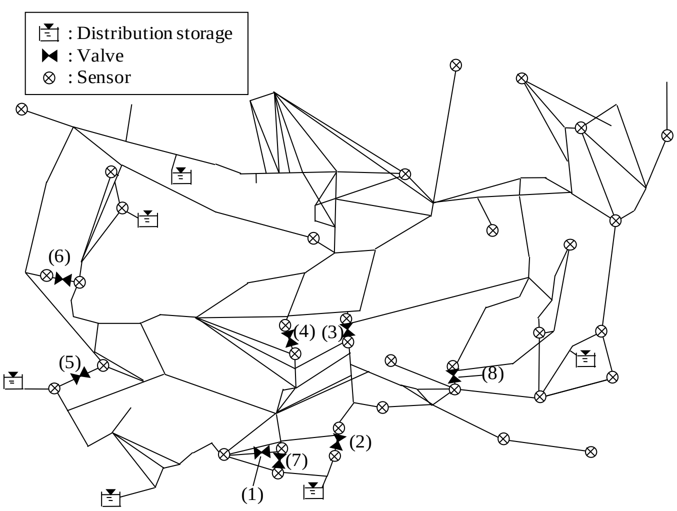


Fig.2 Position of valves and sensors on water distribution pipe network

Determining the structure of the overlapping decomposition means determining the range covered by each controller, which is determined based on the centralized control sensitivity matrix $\frac{\partial H}{\partial u}$.

However, the sensitivity matrix changes in response to the size of the demand, so the sensitivity matrix used to determine the range of the controllers must address demand fluctuation. If sensitivity matrices are drawn up for each demand level when the demand fluctuates as shown in Fig.3, the shapes of the sensitivity matrices are similar even though the demand rises and falls. From this, the mean of the sensitivity matrices created for each demand level is taken as the sensitivity matrix for overlapping decomposition, H^*.

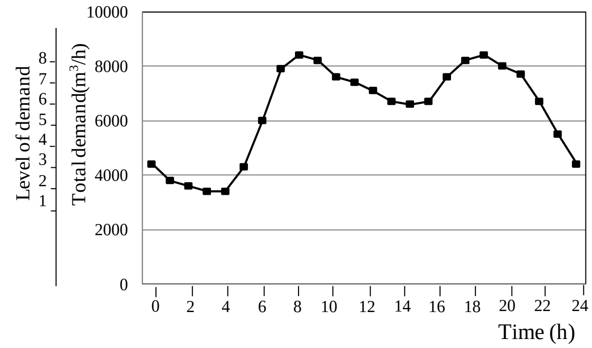
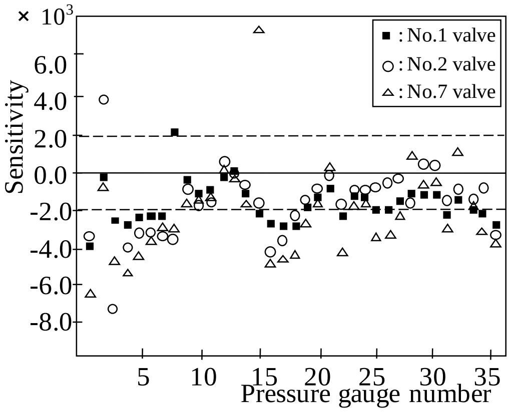


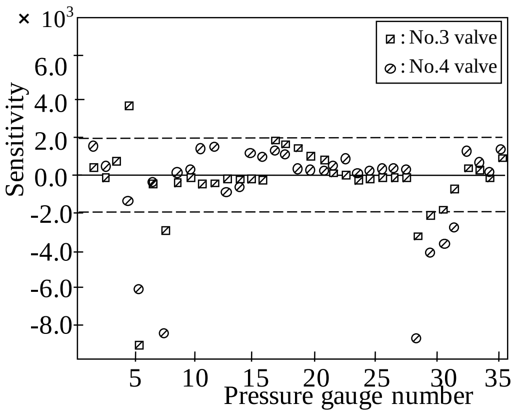
Fig.3 Time varying demand

Based on H^*, we first consider which controller shall cover which operating points. One possibility is a structure in which one operating point is covered by one controller (each controller has multiple inputs and one output), but in order to reduce interference between controllers due to manipulated variables, operating points having structures with similar connection strengths shall be grouped together and covered by one controller. For example, as shown in Fig.4(a), valve numbers 1, 2, and 7 have a similar connection structure to the observation point, and it is evident from Fig.2 that the positional relationship of the valves is adjacent to each other on the upstream side. These valves shall be covered by one controller. However, as shown in Fig.4(d), valve numbers 6 and 8 can be considered to have different connection structures, so they shall be covered by separate controllers. Thus, due to differences in connection structure, the system is controlled by five independent controllers. In summary, the structure will be such that Controller A can operate Valves 1, 2, and 7, Controller B can operate Valves 3 and 4, Controller C can operate Valve 8, Controller D can operate Valve 5, and Controller E can operate Valve 6.

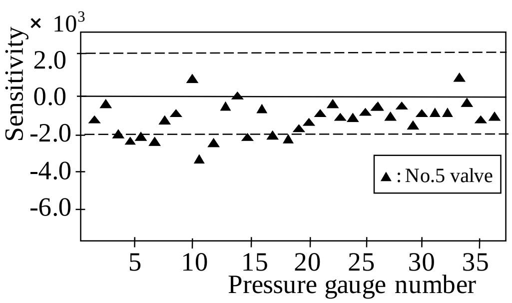
The range of each controller is the region of observation points that have a relatively strong connection relationship to valve operation covered by the controller. Here, the problem is establishing whether there is a relatively strong connection or there is no connection relationship. This is a difficult problem involving the control performance and the processing power of the computer. Here, the degree of overlap of measurement information was taken experimentally as approximately 50%, and, with the idea that any places that are significantly influenced by the operating points covered by each controller will definitely be measured, the boundary for a strong connection was taken as being when the sensitivity matrix element H_{ij} is $|H_{ij}| \geqq 0.002$.



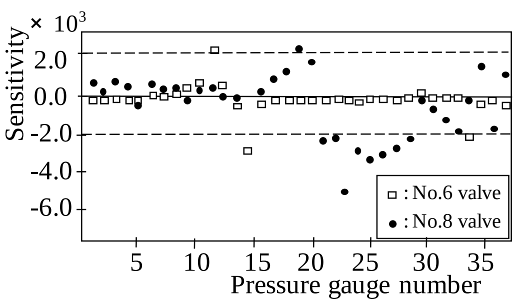
(a) No.1, No.2, No.7 valve



(b) No.3, No.4 valve



(c) No.5 valve



(d) No.6, No.8 valve

Fig.4 Sensitivity of Controlled variables to manipulated variable

The boundary lines are shown in Fig.3. Thus, the information structure of the overlapping control system is determined, as illustrated in Fig. 5.

In addition, in order to carry out a comparative evaluation with a separated control system, the information structure of the separated system is determined as follows. The number of controllers and the problem of which controller covers which valves (operating points) are the same as for the overlapping decomposition structure. In the separated system, measurement information is not overlapped, and so measurement information is input to a given controller. Therefore, observation points are linked by finding the valve (operating point) that most influences each observation point and making this observation point an input to the controller covering that valve. Thus, Fig.6 establishes the information structure of the separated system. When the sensitivity is low, such that $|H_{ij}| < 0.002$, that point shall not be observed, in order to make the conditions uniform with overlapping decomposition.

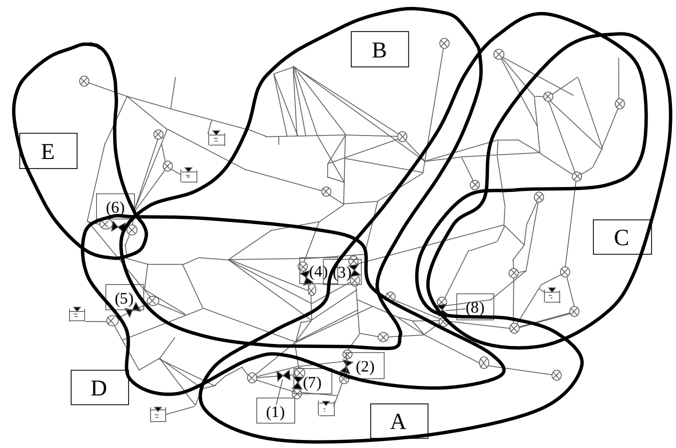


Fig.5 Control configuration using partially shared observation data (Overlapping control structure)

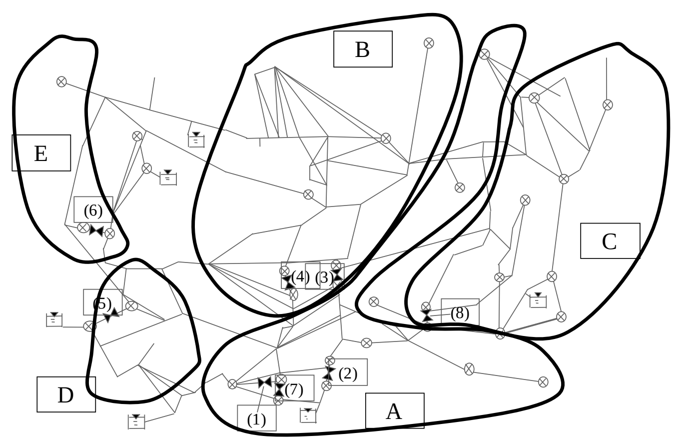


Fig.6 Control configuration using no shared observation data (Separated control structure)

Next, these decentralized control systems are applied to the distribution system, and the control performance, the reliability, etc., are examined.

V. Conclusion

As a measure to reduce the engineering and computer load that pose problems in the control of large-scale distribution systems, in the present paper, we proposed an overlapping adaptive method in which overlapping decomposition of the system is carried out and adaptive controllers are placed in each decomposed subsystem. Overlapping decomposition was shown to be the best method for decomposing the system in order to implement the proposed method. The pipe network in problem has a structure in which the sensitivity of measured variables z to manipulated variables u includes strong nonlinear relationships and disjoint connections. In other words, the input-output structure of the control object H is a nonlinear tightly coupled system that includes partial disjoint connections. By representing the characteristics of this pipe network structure as shown in Equation (11), a mathematical analysis of convergence was prepared. In addition, as shown in Figs.1(a), 1(b), and 1(c), the information structures of centralized control, overlapping control, and separated control were defined. In particular, in the information structure of overlapping control, measurement information is overlapped and measured by multiple controllers. The same linear control strategy was used to study convergence to the optimum point $z_0 = H(u^*)$ in these three control structures. The results showed that, even though the linear control strategy was the same, due to differences in measurement information, the control structures can be ranked in order from high to low convergence as centralized control, overlapping control, and separated control. In other words, it was shown that overlapping control has an advantage over separated control from the perspective of convergence.

Next, in regard to the control method installed in the decentralized controllers, a pipe network adaptive control strategy in which control is carried out by adaptively successively estimating control gains based on a linearized adaptive control model was demonstrated. Control gain G_i, which should be estimated by controller i, is obtained by successive estimation of target process parameter H_i covered by controller i. However, it is not possible to obtain information for manipulated variables covered by other controllers, and so it is necessary to estimate an interference term c_i between subsystems that are not explained by the manipulated variables covered by the controller in problem. Therefore, controller i estimates process parameter H_i from measurement information and manipulated variable information that is available to controller i (in other words, controller i estimates control gain G_i), and controller i also estimates the interference term c_i. This is the control strategy of the adaptive controllers in charge of the decomposed subsystems.

Furthermore, when implementing overlapping control or separated control in a real pipe network, determining the range covered by each controller is essential. The range of each controller in overlapping control was examined using a centralized control sensitivity matrix, which holds all information. As a result, observation points showing the sensitivity matrix element values above a certain specified numerical value were taken to be observation points with a relatively strong connection relationship to valve operation covered by the controller (in other words, the range). This specified numerical value is determined appropriately through engineering by assessing in detail the numerical values of the sensitivity matrix. In separated control, measurement information does not overlap, so that the range of each controller was determined by combining observation points in such a way that a given observation point becomes an input to the controller that is in charge of the operating point that most influences this given observation point.

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