Comparison of fuzzy BK-product and A* search algorithm for optimal path finding in unsupervised underwater environment

Arif Reza Anwary

Abstract**—**The article addresses comparison of fuzzy BK-product approach with A* search algorithm for optimal path finding of the autonomous underwater vehicles (AUVs). The technique is designed to accomplish two missions: obstacle avoidance using ARTMAP and fuzzy BK-product and comparison of optimal path finding with A* search algorithm. In the first mission, underwater obstacle avoidance technique has already implemented [1]. This paper mainly focuses on optimal path finding using fuzzy BK-product and A* search algorithm. AUV gets information about the surroundings through active sonar sensors. An on-line reinforcement learning method is introduced to adapt the performance of the fuzzy units continuously to any changes in the environment and make decision for the optimal path from source to destination. Fuzzy BK-product approach and hardware-in-the-loop simulations have been developed to verify the effectiveness of the proposed technique.

Keywords**—**Fuzzy relation, BK-product, path planning, obstacle avoidance, Autonomous underwater vehicles (AUVs), A* search algorithm

I. INTRODUCTION

AUV possesses obstacle avoidance using fuzzy BK-product carrying out intellectual roles such as decision and action. Many existing systems use global path planning and local path planning to find a safe and reasonable path to reach destination. Adaptive Learning Theory (ART) of neural networks model fast, on-line, and stable categorical learning and recognition system. The default ARTMAP algorithm [2] is a supervised learning version of ART for representing discrete categories using real-valued features.

Among the papers that describe navigation and guiding system for AUVs, a motion planning algorithm based on an annotated map; the path planner retrieves an old, matching, route from the database and modifies it to suit the new situation. This method requires *a-priori* information on the environment and off-line computations that presents [3]. A hybrid controller for AUVs and the controller is composed of three levels: first, discrete, strategic level that labels the current vehicle state and the two, continuous, execution level and tactical level. These implement low-level functions that are respectively synchronous and asynchronous with respect to time that is presented in [4]. An elliptical virtual force field (VFF) is used to command a bottom-following behavior to the Bentic Explorer that is presented [5]. Another presentation focuses on a navigation and guiding system developed for cable inspection that presented in [6]. An on-line 3-D exploring algorithm that generates a mosaicked image of the ocean floor, presented in [7].

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Fuzzy BK-product approach for optimal path finding must allow accomplishment under the following three design constraints. Those are a constant speed of AUV, strength and direction of ocean current should be known and the motion trajectory of AUV should contain the nonholonomic constraints that derived from the actuating system of AUV.

II. FUZZY RELATIONAL METHOD OF BANDLER AND KOHOUT

Bandler and Kohout were the first to introduce the application of fuzzy relational methods to knowledge representation and this method became known as BK-products nowadays. They introduced special relational compositions called the triangle product and square product. BK-products have been applied, as a powerful computational tool, in many fields such as computer protection, AI, medicine, information retrieval, handwriting classification, urban studies, investment, and control [8].

The idea of producing fuzzy relational composition was expanded by Bandler and Kohout in 1977 when they introduced, for the first time, special relational compositions called the triangle product and square product [9,14]. The triangle and square products were named after their inventors and became known as BK-products. This section outlines briefly the basic definitions of BK-products. The mathematical definitions of three families of fuzzy products, namely of triangle sub-product $x(R \triangleleft S)z$, triangle super-product $x(R \triangleright S)z$, and square product $x(R \square S)z$. In the definitions of BK-products that follow, R_{ij}, S_{jk} represent the fuzzy degrees to which the respective statements x_iRy_j, y_jSz_k are true.

Fuzzy logic formulas define the products in the logic notation form. This is advantageous computationally, because finite relations can be expressed and manipulated by a computer in the matrix form. The logical symbols for the logic connectives AND, OR, both implications and the equivalence in the above formulas represent the connectives of some many-valued logic, chosen according to the properties of the products required. The details on choosing the appropriate many valued connectives are discussed in the references [15,16].

BK-relational product can be used to compare and further analyze relational structures. Let R be a relation from X to Y where X is the set of objects and Y is the set of properties. Then R_{xy} is the degree to which a respondent assigns object x to property y. On the other hand, R^T is a relation from Y to X where $R^T = R_{yx}$ is the degree to which a respondent assigns to property y to object x. By composing the relation R with its transposed relation R^T, the fuzzy relational sub-triangle product $R \lhd R^T$ yields an object-object relation over properties by applying a fuzzy implication operator. The product $(R \lhd R^T)_{ij}$ gives the degree to which object x_i implies object x_j based on how a respondent applied both objects to the properties.

The definition of the sub-triangle product is shown in (1) which is used for calculation in this study.

$$x_i(R \lhd S)z_j = \frac{1}{n}\sum_{y \in Y} \min(1, 1 - x_i R(y) + S z_j(y)) \quad \text{--------(1)}$$

In addition, alpha cut (α-cut) and Hasse diagram are two important features in this method. The α-cut transforms a fuzzy relation into a crisp relation, which is represented as a matrix. The Hasse diagram is a useful tool, which completely describes the partial order among the elements of the crisp relational matrix by a Hasse diagram structure [9,10].

III. Digital Image Processing

The imaginary used to detect the obstacles in underwater-unsupervised areas on image processing of gray level based system. Three images were taken from the high-resolution camera with 300dpi. We subsequently examined two methods for filtering the image, each making use of the Red-Green-Blue (RGB) elements of the color image. Instead of the more standard principal components analysis, we have devised a more rigorous method for RGB images which creates the new elements based on the RGB values as follows:

Gray = Red * 0.3 + Green * 0.59 + Blue * 0.11

Using the equation we convert RGB image to gray scale level image. Based on the intensity level of gray color image we find the major and minor components of the image and we do filtering for removing the low noise in the image.

To reduce image noise, one technique is called 'Nova Sharpen' that allows sharpening of blurry images and produces an amazing crispness. The additional Auto-balance instrument of Cleanerzoomer is able to balance colour, levels and gamma of the image greatly increasing its realism.
Dilation and erosion methods have been applied to image for reducing noise from the image.

(a)

(b)

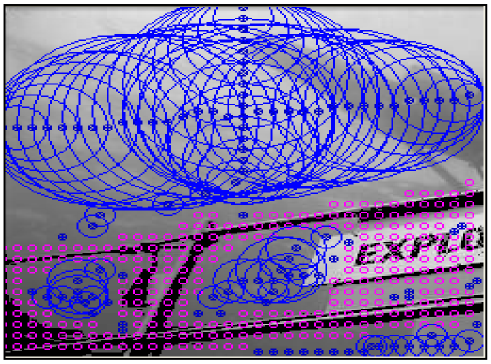
Figure 1: (a) gray level image and (b) normalized image.

For filtering the image, we use dilation and erosion method for constructing elements of morphological image processing. The language and theory of mathematical morphology often presented a dual view of binary images, as a binary image can be viewed as a bivalued function of x and y.

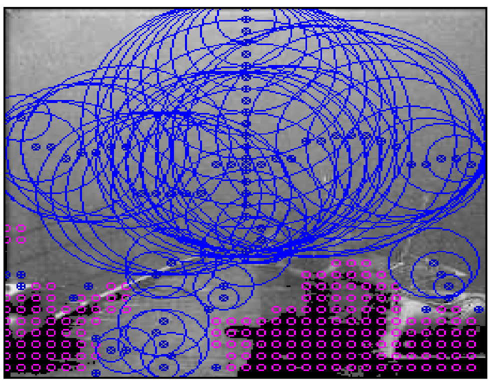
IV. ARTMAP and BK-Subtriangle Products for Obstacle Avoidance

The input matrices (for learning and checking) consist of segmented image information of parameters (x), with the random column being the target output. Here the hard-limit transfer function has used for activation function. So the input space is the space of segmented image and it finds out the major and minor components from the image. Categorization with the ART is performed in three stages: category choice, category match (vigilance test) and learning.

Application of ARTMAP to underwater situation shows in the figure 2. The side of a ship deck and ARTMAP figure out the safety going regions in figure 2(a), the foreword image of a ship underwater and ARTMAP shows the safety ways shows in figure 2(b). Using ARTMAP the information of the underwater can be stored in the memory. It provides radius and direction of the safety path in real time situation. After getting the information first time, weight value will control the parameters and do not need to learn the system all the time. If any new obstacle takes place, the weight value will be changed according to the position of the obstacle. After learning the unstructured underwater situation, mission controller will store the information to the database for next iteration and all other information will be stored to the database as it could have back tracking facility in future work. Based on the information of ARTMAP, fuzzy logic BK-product will decide (Figure 4) the optimal direction based on the radius and distance to the destination that has implemented [1].



(a)



(b)

Figure 2. (a) Side view of a ship deck,(b) Front view of a ship

Suppose that the obstacle avoidance distance can be divided into different portions. All these values will come from ARTMAP and those are the size of the possible directions and distance to goal. Whenever obstacle is detected by ARTMAP, obstacles existence can be identified in the picture. Assume that the distance range can be divided into several sections forming the set S, which are considered as candidates of successive heading. A property set P describes the possibility of AUVs surrounding length of real time environment. The fuzzy rule bases and membership function for the corresponding property can be estimated subjectively by the expert knowledge. With the set of the candidate $S = \{s_1, s_2, ..., s_i\}$ and the set of environmental properties $P = \{p_1, p_2\}$, the relation R is built as (2). The elements r_{ij} of this relation mean the possibility the section s_i can be characterized by the property p_j. The value of r_{ij} is calculated using the rule bases along with the membership functions.

$$R = S \times P = \begin{array}{cc} \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \\ \vdots & \vdots \\ r_{i1} & r_{i2} \end{bmatrix} & \begin{array}{c} s_1 \\ s_2 \\ \vdots \\ s_i \end{array} \\ \begin{array}{cc} p_1 & p_2 \end{array} & \end{array} \quad \text{-----------------(2)}$$

In the next step, a new fuzzy relation T is computed by using sub-triangle product $\triangleleft$ to fuzzy relation R and transposed relation of R. The fuzzy relation T as shown in (3) is the product relation between candidate set S that means the degree of implication among elements of candidate set.

$$T = R \triangleleft R^T = \begin{array}{cc} \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1i} \\ t_{21} & t_{22} & \cdots & t_{2i} \\ \vdots & \vdots & \ddots & \vdots \\ t_{i1} & t_{i2} & \cdots & t_{ii} \end{bmatrix} & \begin{array}{c} s_1 \\ s_2 \\ \vdots \\ s_i \end{array} \\ \begin{array}{cccc} s_1 & s_2 & \cdots & s_i \end{array} & \end{array} \quad \text{--------------(3)}$$

Then, the α-cut is applied to fuzzy relation T in order to transform into crisp relation as shown in (4). It is important to select a reasonable α-cut value because the hierarchical structure of candidate set depends on an applied α-cut. α-cut value depends on the system and for this simulation we choose proper α-cut according to the value of T. Finally, we draw the Hasse diagram, which completely describes a partial order among elements of candidate set, that is to say, a hierarchical structure among the elements of candidate set with respect to the optimality and efficiency. Select then the top node of the Hasse diagram as the successive heading direction of AUVs.

$$R_\alpha = \alpha\_cut(T, \alpha) = \begin{array}{cc} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1i} \\ a_{21} & a_{22} & \cdots & a_{2i} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ii} \end{bmatrix} & \begin{array}{c} s_1 \\ s_2 \\ \vdots \\ s_i \end{array} \\ \begin{array}{cccc} s_1 & s_2 & \cdots & s_i \end{array} & \end{array} \quad \text{----------(4)}$$

According to our AUV, the set $S=\{s_1, s_2, s_3, ..., s_i\}$,which are considered as candidates of successive heading of circles. Radius and Distance (angular displacement) to goal forming represents the set $P=\{p_1, p_2\}$. The Set S and Set P come after learning the environment using unsupervised ARTMAP. These inputs will feed to fuzzy triangular sub-product. The equation becomes

$$R = S \times P = \begin{array}{cc} \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \\ \vdots & \vdots \\ r_{i1} & r_{i2} \end{bmatrix} & \begin{array}{c} s_1 \\ s_2 \\ \vdots \\ s_i \end{array} \\ \begin{array}{cc} p_1 & p_2 \end{array} & \end{array} \quad \text{-----------(5)}$$

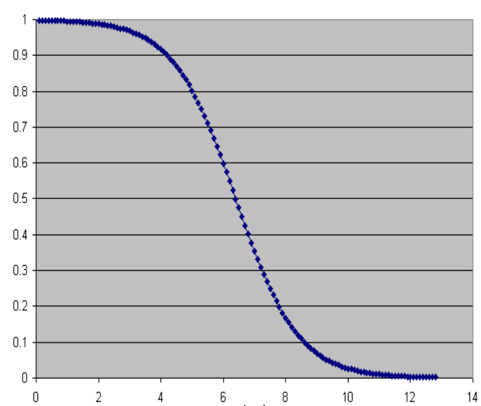
Assume that the radius of circle can be varied into different sections forming the set $S=\{s_1, s_2, s_3, \ldots, s_i\}$,which are considered as candidates of successive heading. This follows the membership function of equation (6), where the value of X is radius value of the circle. Here the variable "l" depends on the maximum value of the radius.

Actually, we determine the two fuzzy properties radius of the circle and angular displacement which is the distance to goal forming the set $P=\{p_1, p_2\}$.This follows the membership function of equation (7), where the value of Y denotes the angular displacement. Here the value of depends on the maximum value of distance. So the fuzzy BK-product will choose the high radius circle with minimum distance to the goal position and it will try to make the distance angel near to zero. In the context of underwater obstacle avoidance of

$$\mu_l(x) = 1 - \frac{1}{1+e^{(l/2-x)}} \qquad \text{---------(6)}$$

$$\mu_\theta(y) = \frac{1}{1+e^{(\theta/2-y)}} \qquad \text{---------(7)}$$

So using the equation (6) and (7), we can draw the "Membership function of Safety Degree" and "Membership function of Angular Degree". The membership function shows figure 4(a) and 4(b) that if the angular distance is low and radius of the circle is high, it is safe and optimal for avoiding the obstacle.



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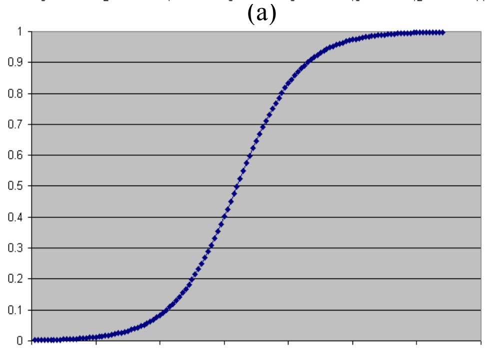


(b)

Figure 3. (a) Membership function of Safety Degree of Radius and (b) Membership function of Angular Degree

This graph drawing considered after taking a scaling function respect to linear threshold value. For the angular displacement we consider about 180^0 of the forward distance, as the AUV cannot move backward direction. The maximum value for radius is the half of the picture value, which depends on camera quality. This value also has taken with a scaling function of linear threshold. Using the equation (6) we have the safety property of radius. Safety property of radius can be defined as following.

Table 1. Safety Property of Radius

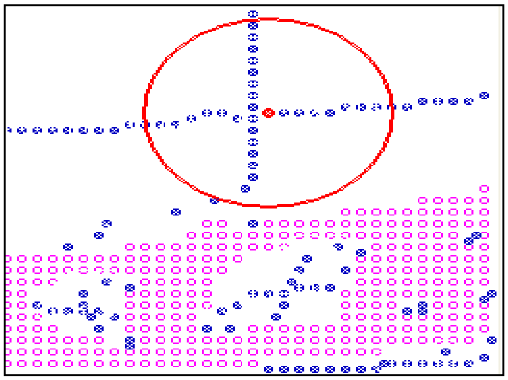
Degree	Situation	Value of x
Very Safe	IF: S_i is not adjacent to any obstacle	THEN: x = 1
Safe	IF: S_i is adjacent to obstacle	THEN: x = 6
Risk	IF: S_i is between the obstacle	THEN: x = 8
Very Risk	IF: S_i contains obstacle	THEN: x = 10

The equation (6) represents the membership function of Angular Degree. Using the equation (7) we have the angular property. So the angular property can be defined as following.

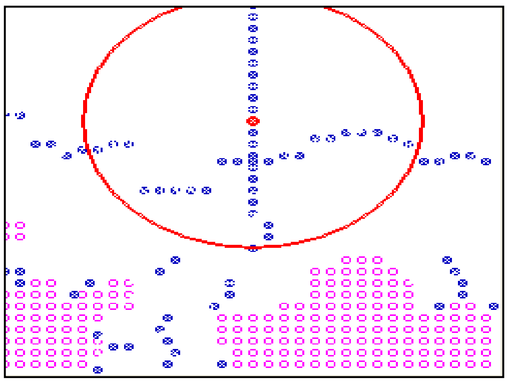
Table 2. Angular Property

Degree	Situation	Value of y
Large	IF: S_i 's angle to destination is very large'	THEN: y = 1
Medium	IF: S_i 's angle is medium	THEN: y = 6
Low	IF: S_i 's angle is low	THEN: y = 8
Very Low	IF: S_i 's angle is very low	THEN: y = 10

An initial investigation revealed that knowledge representation in a traditional expert system is difficult because of numerous problems of knowledge acquisition. Problem solving is seldom a purely logical process based on the application of fuzzy BK-product. The ARTMAP will learn the system and generate the possible ways of safety directions. Given the availability of information extracted in the BK-product, it would be advantageous to be able to learn from it; unfortunately BK-product does not generally offer such a learning mechanism. On the other hand, an ARTMAP is able to learn from the noisy data.



(a)



(b)

Figure 4. The safety direction after fuzzy BK-product respect to Figure 2.

The input matrices (for learning and checking) consist of segmented image information of parameters (x), with the random column being the target output. Here the hard-limit transfer function has used for activation function. So the input space is the space of segmented image and it finds out the major and minor components from the image. Categorization with the ART is performed in three stages: category choice, category match (vigilance test) and learning. In the category choice stage, a choice function is calculated for the current pattern *I* and each existing category

$$T_j = \frac{\|I \wedge w_j\|}{\alpha + \|w_j\|} \quad \text{----------------(8)}$$

where ^ is the AND operation $(X \wedge Y)_i = \min(x_i, y_i)$, $\alpha > 0$ is a choice parameter 2 and the norm is L_1(L_1-norm |x| of a vector x). The chosen category is the one achieving the highest value of the choice function. When a category *J* is chosen, a hypothesis test called the vigilance test is performed in order to measure the category match to the pattern. If the match function exceeds the vigilance parameter $\rho \in [0, 1]$

$$\frac{\|I \wedge w_j\|}{\|I\|} \geq \rho \quad \text{-----------------(9)}$$

Then the chosen category is said to win (match) and learning is performed. Otherwise, the chosen category is removed from the search by forcing the value of T_J to zero for the rest of this pattern presentation. As a result, a new category maximizing the choice function (8) is chosen and the process continues until a chosen category satisfies the vigilance test (9). If none of the existing categories meets the vigilance test, a new category is formed and learning for this category is performed without a vigilance test. Either way, learning in the fuzzy ART is accomplished by updating the weight vector of the winning (or new) category according to

$$w_j^{new} = \beta(I \wedge w_j^{old}) + (1-\beta)w_j^{old} \quad \text{-------------(10)}$$

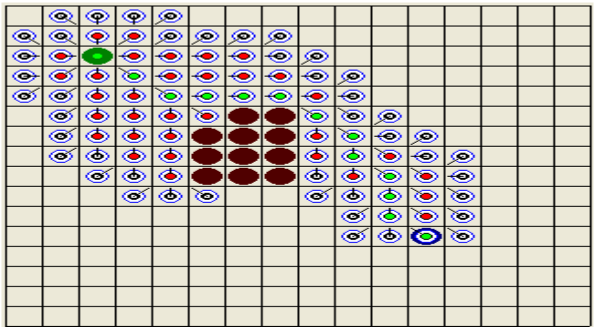
where $\beta \in [0, 1]$ is the learning rate and $\beta = 1$ defines fast learning. Note that the vigilance parameter controls the similarity required between the chosen category and the input pattern (10) in order to control learning. For the simulation assuming that the goal position is located in the center of the picture (128x128). Lowering the vigilance parameter provides broader generalization (large categories) and vice versa. The data from the input pictures based on ARTMAP learning procedure.[5]

V. The A* Algorithm and Fuzzy BK-Product Decision Function

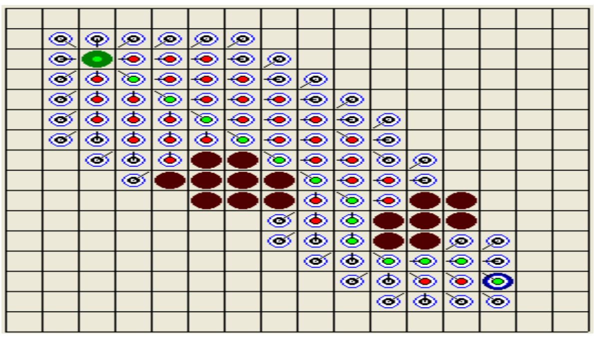
The A* algorithm is used to find heuristic pathways with low cost and optimal solution. Some of the reasons for using it are because it can be interrupted and resumed later and it can handle terrain with different traversal difficulty [11,12]. But we compare here A* search algorithm with Fuzzy logic based decision path for online searching to reach the destination. Different experimental results shows that A* search algorithm sometime does not give good result in the case of underwater path planning for AUV. It needs sensorial data to calculate the successors adjacent. Using Fuzzy-BK product decision function, the path for the AUV shows the better result for unstructured underwater environment.

To find the shortest path, A* algorithm starts from a point, START (source), to another point, GOAL (destination). A* keeps a list of all the possible next steps, called the OPEN list. It then chooses a next step that is most likely to lead us to the goal in the minimum time. In order to accomplish this we need to have a heuristic that will determine "most likely". Once that step has been chosen, it is moved to the CLOSED list.

So using the A* search algorithm we can see the results of different situations for path planning with obstacle avoidance for underwater AUV.



(a)



(b)

Figure 5. The result of A* search algorithm for obstacles in different situations

A* search algorithm result for one obstacle show in figure 5(a) results and two obstacles in 5(b). When A* terminates its search, found a path whose actual cost is lower than the estimated cost of any path through any open node. But since those estimates are optimistic among all other paths, A* can safely ignore those nodes [13]. But in the case of fuzzy based path from source to destination shows the online system. It works well in unknown territory with camera to find out the possible directions without obstacle avoidance.

When one wants to address the problem of obstacle avoidance in the underwater-unstructured environment, the main problem encountered is the extraction of information from the input data to create a representation of the environment that is as close as possible to the "ground truth" scene and can be interpreted in terms that are suitable for computation. The system we have designed (Figure 4) is modular in nature. So the purpose of each module handles different needs within the same framework.

The camera captures the pictures from underwater situation and it sends that data to the computer for analyzing and computation. The module sends images frame by frame continuously and dynamically. The first image learns and finds out the possible ways to go after avoiding potential obstacles and their features (position, moments, area) are computed. These features are used later to discard false alarms and track the obstacles and the vehicle.

Next module collects the possible ways to reach the destination from ARTMAP and using fuzzy BK-product, it chooses the best and optimal direction to reach the destinations avoiding collision of the obstacle with the AUV.

Using fuzzy BK-product to find the shortest path (starting point is called Source, and the ending point is called Destination), the system keeps a list of paths from source to destination directly without checking obstacle at the first step, called the OPEN list. It then chooses a next step after checking obstacle in front of the vehicle using the help of ARTMAP that is most likely to lead to the goal in the minimum time. Once that step has been chosen, it is moved to the OPEN list. The AUV will follow the OPEN list to reach the destination safely.

The algorithm becomes:

```
Start using nodes Source to Destination
Add Source to OPEN list in a queue
   while OPEN not empty from the queue
      if obstacle then call ARTMAP
            learn environment;
            find obstacle position
            find optimal direction using Fuzzy BK product
      end if
      if n is Destination then return path
      move AUV to destination direction using OPEN list
   end while
if we get to here, then there is No Solution
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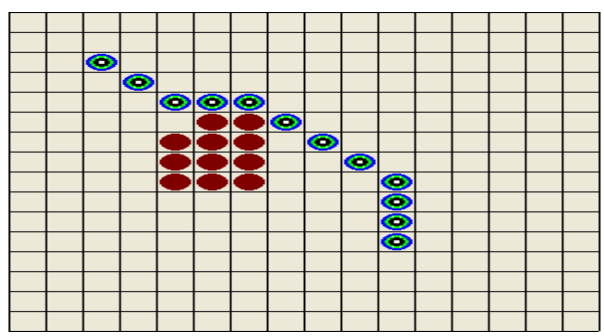
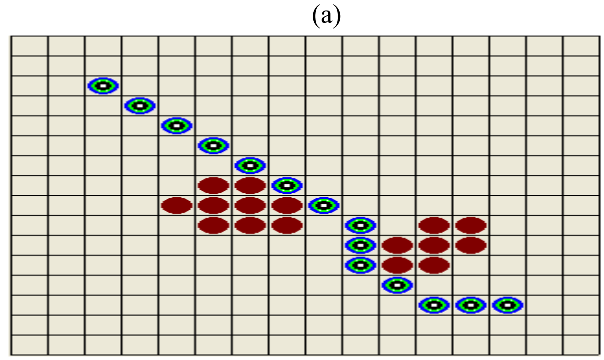


(a)



(b)

Figure 6.The result of source to destination using fuzzy BK-product decision function

In the figure 6, the result of the path using fuzzy BK-product decision function for one obstacle show in figure 6(a), and result for two obstacles in figure 6(b). If we take a close look at figure 5(a) and figure 6(a), the way point for A* search algorithm takes thirteen points and in the case of fuzzy decision function it takes only twelve points. But it is not same always to ensure the minimum distance using fuzzy decision function. In the case of successors adjacent we can conclude that fuzzy decision function takes low calculation with optimistic solutions and A* search algorithm takes time and sensors data to find the optimal path. For underwater unstructured environment A* search algorithm will not perform well as it is offline searching technique.

Table 3: Experimental result of A* search Algorithm

Array Size	Number of Obstacle(s)	Required Time for A* (milliseconds)	A* Search Algorithm Waypoints
16x16	1	0.094	13
16x16	2	0.125	16
16x16	3	0.125	17
32x32	1	1.187	34
32x32	2	1.156	35
32x32	3	1.719	41
64x64	1	5.562	74
64x64	2	6.844	71
64x64	3	5.184	76

Table 4: Experimental result of Fuzzy-BK product decision function

Array Size	Number of Obstacle(s)	Required Time for Fuzzy-BK (milliseconds)	Fuzzy-BK product Decision Function Waypoints
16x16	1	0.001	12
16x16	2	0.015	15
16x16	3	0.016	17
32x32	1	0.016	34
32x32	2	0.016	36
32x32	3	0.016	43
64x64	1	0.016	76
64x64	2	0.016	73
64x64	3	0.016	79

The performance of the A* Algorithm is compared to results obtained by fuzzy BK-product decision function based on x and y coordinate. It takes low time to compute the different waypoints for fuzzy function and A* search algorithm takes longer time based on the array size. The waypoint varies according to number of increased array size comparing with Fuzzy BK-product decision function and A* algorithm. There exist theoretical and technical challenges in this project. The concepts of convergence and optimality for moving AUVs to target position problems invoke the theoretical challenges. Indeed, it will take much time for 3D underwater environment to compute path planning using A* search algorithm that is shown in table 3 and table 4. BK-product helps to find way points within low memory in real underwater situation as it takes images from camera with x and y coordinate.

VI. Conclusion

ARTMAP and fuzzy BK-product based obstacle avoidance technique and comparison of fuzzy BK-product and A* search algorithm for optimal path finding for underwater-unsupervised environment have been fabricated and tested with good results for autonomous underwater vehicles. This paper describes the technique of controlling autonomous underwater vehicles which makes the possible integration of fuzzy decision for optimal path finding. This technique is tested under real conditions. We think that the proposed strategy is an effective approach. Another important result is the effective intelligent motion capability of the proposed scheme. With the help of its “brain” (fuzzy-decision, which makes decision about any action of the vehicle), AUV makes list of ways to reach destination autonomously and stably chooses the optimal direction to reach its target avoiding unknown and/or moving obstacles with optimal path in unstructured underwater areas.

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