

# Broadcasting and Routing Algorithms for the Extended OTIS-Cube Network

Jehad A. Al-Sadi

**Abstract**— This paper introduces a routing algorithm for the Extended OTIS- $n$ -Cube Networks. The recently proposed network has many good topological features such as regular degree, semantic structure, low diameter, and ability to embed graphs and cycles. Broadcasting approaches such as constructing a Hamiltonian cycle, Spanning Tree, and One to all broadcasting are important aspects for any topology to embed due to the importance of broadcast messages within networks. This paper also presents such approaches for the extended OTIS- $n$ -Cube Interconnection network. Examples are presented for different network sizes to show that broadcasting approaches for the Extended OTIS- $n$ -Cube give better performance compared to similar approaches for the OTIS- $n$ -Cube.

**Keywords**— Interconnection Networks, OTIS- $n$ -Cube, Broadcasting, Routing Algorithm.

## I. INTRODUCTION

THE binary  $n$ -cube has been one of the most popular network topologies for multicomputers due to its attractive topological properties, e.g. regular structure, low diameter, and ability to exploit communication locality[1]. Several experimental and commercial systems have been built using the factor cube network including the NCUBE-2 [2], Intel iPSC [3], Cosmic Cube [4], and SGI Origin 2000 multiprocessor [5].

In the last decade, there has been an increasing interest in a class of interconnection networks called Optical Transpose Interconnection Systems “OTIS-networks” [6] – [9]. Marsden *et al* were the first to propose the OTIS-networks [10]. Extensive studies and modeling results for the OTIS have been reported in [11] – [14]. The achievable terabit throughput at a reasonable cost makes the OTIS a strong competitor to the electronic alternatives [10], [15] - [17]. These encouraging findings prompt the need for further testing of the suitability of the OTIS for real-world parallel applications.

A number of computer architectures have been proposed in which the OTIS was used to connect different processors [18]. Krishnamoorthy et al [12] have shown that the power consumption is minimized and the bandwidth rate is maximized when the OTIS computer is partitioned into  $N$

groups of  $N$  processors each. [12, 19]. Furthermore, the advantage of using the OTIS as optoelectronic architecture lies in its ability to maneuver the fact that free space optical communication is superior in terms of speed and power consumption when the connection distance is more than few millimeters [12]. In the OTIS, shorter (intra-chip) communication is realized by electronic interconnects while longer (inter-chip) communication is realized by free space interconnects.

OTIS technology processors are partitioned into groups, where each group is realized on a separate chip with electronic inter-processor connects. Processors on separate chips are interconnected through free space interconnects. The philosophy behind this separation is to utilize the benefits of both the optical and electronic technologies.

Processors within a group are connected by a certain interconnecting topology, while transposing group and processor indexes achieve inter-group links. Using  $n$ -cube as a factor network will yield the OTIS- $n$ -Cube in denoting this network.

OTIS- $n$ -Cube is basically constructed by "multiplying" a cube topology by itself. The set of vertices is equal to the Cartesian product on the set of vertices in the factor cube network. The set of edges  $E$  in the OTIS- $n$ -Cube consists of two subsets, one is from the factor cube, called *cube*-type edges, and the other subset contains the *transpose* edges. The OTIS approach suggests implementing *cube*-type edges by electronic links since they involve intra-chip short links and implementing transpose edges by free space optics. Throughout this paper the terms “*electronic move*” and the “*OTIS move*” (or “*optical move*”) will be used to refer to data transmission based on electronic and optical technologies, respectively.

Although the OTIS- $n$ -Cube network has many attractive topological properties but it suffers from having limited optical links between the different groups. When source and destination nodes are in two different groups, the fact that only one optical link connects two distinguished groups directly create a congestion problem to most of the shortest paths that have to pass through this particular optical link. Furthermore, alternative paths are too long compared to the

short path because they have to be routed via a third group which required passing via two optical links in addition to the electronic moves in each group to reach the destination.

The Extended OTIS- $n$ -Cube is a recently proposed interconnection network based on the "OTIS- $n$ -Cube" network [20]. This new topology has many attractive properties such as the regular degree, the small diameter, embedding structure nature, etc. The Extended OTIS- $n$ -Cube network outperforms the OTIS- $n$ -Cube in many feature including semantic structure, regularity, smaller diameter, and other exceptional properties [20].

Broadcasting refers to a method of transferring a message to all other nodes simultaneously. Broadcast operation is one of the most fundamental services utilized frequently by other communication mechanisms in any network topology. Supporting efficient broadcast operation is therefore very crucial for all networks including the Extended OTIS- $n$ -Cube. Embedding of topologies with regular structure and also irregular structure has been broadly investigated in the literature, e.g [18, 19, 21, 22]. Embedding structures and other topologies is one of the key features of interest in interconnection networks. The load of an embedding is the maximum number of nodes in a graph assigned to any node in the embedded graph. We are interested in this research in investigating routing algorithms and broadcasting techniques including embedding a Hamiltonian cycle and blanched trees approaches [23].

In the mathematical field of graph theory, a Hamiltonian path is a path in an undirected graph which visits each node exactly once. A Hamiltonian cycle is a cycle in an undirected graph which visits each node exactly once and also returns to the starting node. Determining whether such paths and cycles exist in graphs is the Hamiltonian path problem [21, 22, 24].

The Hamiltonian path seeks whether there is a route in a directed network from a beginning node to an ending node, visiting each node exactly once. The Hamiltonian path problem is NP complete, achieving astonishing computational complexity. This challenge has inspired researchers to broaden the definition of computer computations. The Hamiltonian problem arises in many real world applications including DNA applications [22].

This paper proposes a theoretical study on the routing properties in general and broadcasting techniques in specific for the Extended OTIS- $n$ -Cube due to its attractive properties. Section 2 presents notations and preliminary definitions. Details of embedding a Hamiltonian cycle in the Extended OTIS- $n$ -Cube topology will be discussed in section 3. Section 4 concludes the paper.

## II. PRELIMINARY DEFINITIONS

The  $n$ -dimensional undirected graph binary  $n$ -cube is one of the well known networks which have been used in real life systems [25] – [28].

**Definition 1:** The undirected graph  $n$ -cube with  $2^n$  vertices, representing nodes, which are labelled by the  $2^n$  binary digits of length  $n$ . The binary system consists of two bits; 0 and 1. Two nodes are connected by a direct edge if, and only if, their labels differ in exactly one bit position.

The Extended OTIS- $n$ -Cube is constructed by "multiplying" a cube topology by itself. The vertex set is equal to the *Cartesian* product on the original vertex set in the factor cube network. The initial step is similar to OTIS- $n$ -Cube construction; this is why we name it Extended OTIS- $n$ -Cube.

**Definition 2:** Let  $\langle g_1, p_1 \rangle$  be group and processor addresses of a node in an Extended OTIS- $n$ -Cube labeled as series of bits  $\langle x_n \dots x_2 x_1 \rangle$ ,  $\langle y_n \dots y_2 y_1 \rangle$  consequently where each bit is either 0 or 1. A node  $\langle g_2, p_2 \rangle$  is called an *opposite* of node  $\langle g_1, p_1 \rangle$  if and only if they differ only in the first bit position of  $g_1$  and  $g_2$  labels, and also in the first bit position of  $p_1$  and  $p_2$  labels; they differ only in  $x_1$  and  $y_1$ , e.g. node  $\langle 00, 00 \rangle$  is an opposite node of  $\langle 01, 01 \rangle$ .

**Definition 3:** The two nodes  $\langle g_1, p_1 \rangle$  and  $\langle g_2, p_2 \rangle$  are connected via a transpose edge if and only if  $g_1 = p_2$  and  $g_2 = p_1$ .

The edge set consists of electronic edges from the factor network and two new types of edges called the transpose and opposite edges, both types are considered optical edges. The formal definition of the Extended OTIS- $n$ -Cube is given below.

**Definition 4:** Let  $n$ -cube =  $(V_0, E_0)$  be an undirected graph representing an  $n$ -cube network where  $n$  is the cube degree. The Extended OTIS- $n$ -Cube =  $(V, E)$  network is represented by an undirected graph obtained from  $n$ -cube as follows  $V = \{ \langle g, p \rangle \mid g, p \in V_0 \}$  and  $E = \{ \langle \langle g, p_1 \rangle, \langle g, p_2 \rangle \rangle \mid \langle p_1, p_2 \rangle \in E_0 \} \cup \{ \langle \langle g, p \rangle, \langle p, g \rangle \rangle \mid g, p \in V_0 \} \cup \{ \langle \langle g, g \rangle, \langle p, p \rangle \rangle \mid g, p \in V_0 \cap g \text{ is an opposite of } p \}$

In the Extended OTIS- $n$ -Cube, the address of a node  $u = \langle g, p \rangle$  from  $V$  is composed of two components.

Figure 1 shows a 16 processor Extended OTIS-2-Cube, solid arrows represent transpose edges while dashes arrows represent opposite edges. The notation  $\langle g, p \rangle$  is used to refer to the group and processor addresses respectively, two nodes  $\langle g_1, p_1 \rangle$  and  $\langle g_2, p_2 \rangle$  are connected by a direct edge if one of the following cases occurs:

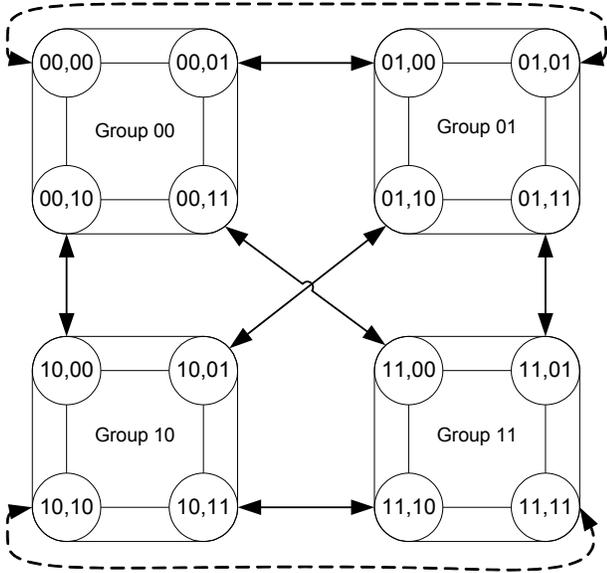


Fig. 1 16-processor Extended OTIS-2-Cube

1. If  $g_1 = g_2$  and  $(p_1, p_2) \in E_0$  where  $E_0$  is the set of edges in  $n$ -cube network, in this case the two nodes are connected by an electronic edge if their labels differ only by one bit position.
2. If  $g_1 = p_2$  and  $p_1 = g_2$ , in this case the two nodes are connected by a transpose edge.
3. If  $g_1 = p_1$ ,  $g_2 = p_2$ , and  $g_1$  is an opposite of  $g_2$ , then the two nodes are connected by an opposite edge.

### III. THE ROUTING ALGORITHM

In this section we introduce the routing algorithm for the Extended OTIS- $n$ -Cube. First, we introduce some routing topological properties of the Extended OTIS- $n$ -Cube which are needed to show the present the unicast routing and also the broadcasting algorithms.

**Definition 5:** If the cube factor degree is  $n$ , then any node in the Extended OTIS- $n$ -Cube is regular and the node degree is  $n+1$ .

Every node has  $n$  electronic edges based on the properties of the  $n$ -cube factor. Also every node;  $\langle g, p \rangle$ ; has an additional optical edge based on the Extended OTIS- $n$ -Cube topology rule:  $\{(\langle g, p \rangle, \langle p, g \rangle) \mid g, p \in V_0\} \cup \{(\langle g, g \rangle, \langle p, p \rangle) \mid g, p \in V_0 \cap g \text{ is an opposite of } p\}$

so If  $g=p$  then  $\langle g, p \rangle \xrightarrow{O} \langle g_{op}, g_{op} \rangle$  else  $\langle g, p \rangle \xrightarrow{O} \langle p, g \rangle$ .

Since every node has an  $n$  number of electronic, in addition to one optical edge, then by definition the topology is regular.

**Definition 6:** Let  $\langle g_1, p_1 \rangle$  and  $\langle g_2, p_2 \rangle$  be two different nodes in the Extended OTIS- $n$ -Cube. The length of shortest path from the source node  $\langle g_1, p_1 \rangle$  to the destination node  $\langle g_2, p_2 \rangle$  is defined mutually exclusive as in the following order:

$$\text{Length} = \begin{cases} d(p_1, p_2) & \text{if } g_1 = g_2 \\ d(p_1, g_1) + d(g_{1\text{Opposite}}, p_2) + 1 & \text{if } g_1 = g_{2\text{Opposite}} \\ d(p_1, g_1) + d(p_{1\text{Opposite}}, g_2) + 2 & \text{if } p_1 = p_{2\text{Opposite}} \\ d(p_1, p_2) + d(g_1, g_2) & \text{if } g_1 = p_1 \text{ and } g_2 = p_2 \text{ and } d(g_1, g_2) = n \\ \min(d(p_1, g_2) + d(p_2, g_1) + 1, d(p_1, p_2) + d(g_1, g_2) + 2) & \text{Otherwise} \end{cases}$$

Where  $d(p_1, p_2)$  is the number of bit positions differ between  $p_1$  and  $p_2$  labels.

The length of the shortest path between the nodes  $\langle g_1, p_1 \rangle$  and  $\langle g_2, p_2 \rangle$  can be addressed as follows:

-If both nodes are in the same group then the shortest path is guaranteed by generating electronic moves toward the destination;  $d(p_1, p_2)$ .

- If  $g_1 = g_{2\text{op}}$  it means that one optical move is needed to move toward the destination group via a group opposite edge. To reach the destination, some electronic moves might be needed first at one source group to reach  $\langle g_1, g_1 \rangle$ , then one optical move to reach the destination group; finally other electronic moves at the destination group might be needed to reach the destination node.

- If  $p_1 = p_{2\text{op}}$  it means that two optical moves are needed to reach the destination group through an intermediate group equal to  $p_{1\text{op}}$ . This requires some electronic moves to perform the two optical moves, and finally to reach the destination node at optimal distance.

- If  $p_1 = p_2$ ,  $g_1 = g_2$ , and  $d(p_1, p_2) = n$  it means that two optical moves in addition to some electronic moves are needed to reach the destination group through an intermediate group  $g_{1\text{op}}$ . First an opposite move is required to reach  $\langle g_{1\text{op}}, p_{1\text{op}} \rangle$ , then  $n-1$  electronic moves to reach  $\langle g_{1\text{op}}, g_2 \rangle$ , then an optical move to reach  $\langle g_2, g_{1\text{op}} \rangle$ , and finally other  $n-1$  electronic moves to reach the destination node  $\langle g_2, p_2 \rangle$  at optimal distance.

- Otherwise we choose the shortest path based on the factor OTIS moves [29].

Figure 2 shows the unicast routing algorithm where each source node  $\langle g_1, p_2 \rangle$  in the network applies to route a message towards its destination node  $\langle g_2, p_2 \rangle$ .

The new algorithm out perform the previous one introduced in [30] by considering all routing cases including diameter distances where  $g_1 \neq p_2$  and/or  $g_1 \neq p_2$ .

```

Algorithm: Routing(M: message;  $\langle g_1, p_1 \rangle, \langle g_2, p_2 \rangle$ :
node)
/* called by current node  $\langle g_1, p_1 \rangle$  to route the message
M toward
its destination node  $\langle g_2, p_2 \rangle$  */
if  $g_1 = g_2$  and  $p_1 = p_2$  then exit; /* destination reached */
if  $g_1 = g_2$  then route( $\langle g_1, p_1 \rangle, \langle g_2, p_2 \rangle$ ) /* curr & dest. at
the same group */
if  $dist(p_1, p_1) + dist(g_1, g_2) = \text{diameter}$  then move m to  $\langle g_1$ 
Opposite  $p_1$  Opposite  $\rangle$ 
if  $g_1 = g_2$  Opposite then /* curr and dest are in opposite groups
{
if  $p_1 = g_1$  then move m to  $\langle g_1$  Opposite,  $g_1$  Opposite  $\rangle$ 
else if  $p_1 = g_2$  then move m to  $\langle g_2, p_1 \rangle$ 
else route( $\text{Min}(\langle g_1, p_1 \rangle, \langle g_1, g_1 \rangle$  or  $\langle g_1, p_1 \rangle, \langle$ 
 $g_1, g_2 \rangle$ ))
}
if  $p_1 = p_2$  Opposite then /* curr and dest are in opposite
processes */
{ if  $p_1 = g_1$  then move m to  $\langle g_1$  Opposite,  $g_1$  Opposite  $\rangle$ 
else move m to  $\langle p_1, p_2$  Opposite  $\rangle$  }
if  $(dist(p_1, p_2) + dist(g_1, g_2) + 2) < (dist(p_1, g_2) + dist(g_1, p_2) + 1)$ 
then
{ if  $p_1 = p_2$  then move m to  $\langle p_1, g_1 \rangle$ 
else route( $\langle g_1, p_1 \rangle, \langle g_1, p_2 \rangle$ ) }
else
{ if  $p_1 = g_2$  then move m to  $\langle p_1, g_1 \rangle$ 
else route( $\langle g_1, p_1 \rangle, \langle g_1, g_2 \rangle$ ) }
}
End.
Function route( $\langle g_1, p_1 \rangle, \langle g_2, p_2 \rangle$ : node)
{
send M to  $(\langle g_1, p_1^{(i)} \rangle$  )
/* select a preferred neighbor toward the destination
node  $\langle g_2, p_2 \rangle$  */
}

```

Fig. 2 The Unicast Routing Algorithm

The Algorithm checks first whether the source and the destination nodes are in the same group or not. If both nodes are in the same group then the factor cube routing rules are applied by selecting a preferred neighbor to guarantee an optimal routing toward the destination. Otherwise, the algorithm selects a move that leads to make an optical move to reach the destination's group, then, to reach the target destination node.

This routing algorithm checks first the related locations of the source and the destination nodes. If both nodes are in the same group; where the factor network is the  $n$ -Cube; then the algorithm corresponds to a series of electronic moves from  $p_1$  to  $p_2$  along an optimal path in the  $n$ -Cube as generated by the optimal distance routing function. If the source node is located in different group than the destination node's group then there are three cases.

In the first case, if the distance is equal to the diameter then the optimal routing path will contain two optical moves in addition to a set of electronic moves; as explained in the previous section; the algorithm will go through several conditions based on the addresses of source and destination nodes to choose the proper moves to reach the destination node.

In the second case, if the group of destination node is in an opposite group of the source node then the optimal routing path will contain only one optical move in addition to electronic moves. The algorithm will choose an opposite move or a transpose move for this optical move based on the addresses of source and destination nodes to choose the proper moves to reach the destination node.

In the third case, if the process address of destination node is in an opposite process address of the source node then the optimal routing path will contain either one or two optical moves in addition to electronic moves based on the addresses of source and destination nodes, the algorithm will choose the proper direction to reach the destination node as shown in Figure 2.

**Example 1:** Consider the Routing algorithm to route a message from the source node  $\langle 01, 00 \rangle$  to the destination node  $\langle 01, 11 \rangle$  in an Extended OTIS-2-Cube network.

Since both nodes are in the same group; the same factor 2-Cube network; then the algorithm corresponds to a series of electronic moves from process address 00 to 11 along an optimal path in the 2-Cube as generated by the optimal distance routing. These moves are as follows:

$$\langle 01, 00 \rangle \xrightarrow{E} \langle 01, 01 \rangle \xrightarrow{E} \langle 01, 11 \rangle.$$

**Example 2:** Consider routing algorithm to route a message from the source node  $\langle 00, 00 \rangle$  to the destination node  $\langle 01, 11 \rangle$  in an Extended OTIS-2-Cube network.

Since both nodes are in the different groups opposite to each other, then the routing path contains one optical move. The routing path will be  $\langle 00, 00 \rangle \xrightarrow{O} \langle 01, 01 \rangle \xrightarrow{E} \langle 01, 11 \rangle$ .

**Example 3:** Consider routing algorithm to route a message from the source node  $\langle 00, 00 \rangle$  to the destination node  $\langle 11, 11 \rangle$  in an Extended OTIS-2-Cube network.

Since both nodes are in the different groups at the diameter distance, then the routing path contains two optical moves. Based on the routing algorithm, the routing path will be  $\langle 00, 00 \rangle \xrightarrow{O} \langle 01, 01 \rangle \xrightarrow{E} \langle 01, 11 \rangle \xrightarrow{O} \langle 11, 01 \rangle \xrightarrow{E} \langle 11, 11 \rangle$ .

**Example 4:** Consider routing algorithm to route a message from the source node  $\langle 00, 00 \rangle$  to the destination node  $\langle 10, 01 \rangle$  in an Extended OTIS-2-Cube network.

Since both nodes are in the different groups where the process addresses are opposite to each other, then the routing path contains only one optical move. Based on the routing algorithm, the routing path will be  $\langle 00, 00 \rangle \xrightarrow{E} \langle 00, 10 \rangle \xrightarrow{O} \langle 10, 00 \rangle \xrightarrow{E} \langle 10, 01 \rangle$ .

#### IV. BROADCAST ALGORITHMS

In this section, we introduce three broadcast algorithms for the Extended OTIS- $n$ -Cube networks; we also compare such using such algorithms in both Extended OTIS- $n$ -Cube and the OTIS Cube.

##### A. One-to-all Broadcasting

While unicast routing has proved very useful in computer resource sharing, certain distributed computing applications requiring one-to-all communication have been suffering. Broadcast one-to-all communication is the delivery of messages to all destination nodes, while multicast, or multi destination delivery, is the delivery of messages to some specified subset of all the destinations.

Broadcast routing is defined to be the routing procedures by which broadcast is achievable in inherently non broadcast communication networks. Broadcast routing is a special case of multi destination

One-to-all broadcasting is a communication process in which a single source node sends the same message to every other node in the network. It is used in many applications. Examples are Gauss elimination method, and matrix-vector multiplication [31]. An optimal broadcasting algorithm for OTIS-Cube networks is presented in [29]. First we introduce a one to all routing algorithm that can be applied to OTIS Cube:

```

OTIS_Cube One-to-all Broadcasting
{
  Step1:  $\langle g_i, p_j \rangle$  broadcasts  $M$  locally to every node in the
  group  $\langle g_i \rangle$ , where  $\langle g_i, p_j \rangle$  is the source node and  $M$  is the
  message to be broadcasted.

  Step 2: Every node  $\langle g_i, p_k \rangle$ ,  $1 \leq k \leq 2^n, k \neq i$ , sends  $M$  to
   $\langle p_k, g_i \rangle$  over the optical link.

  Step 3: Every node  $\langle p_k, g_i \rangle$ ,  $1 \leq k \leq 2^n, k \neq i$ , broadcasts
   $M$  locally in the  $\langle p_k \rangle$  group.
}

```

Using the above algorithms take  $2n+1$  sequential time units in an OTIS-Cube network. For step 1, sending the message to all local nodes in parallel will take  $n$  time units, the same applies for step3. Step 2 takes one time unit since the message is transferred to the  $2^n - 1$  groups are occurred in parallel at once.

Now we present the following simple algorithm to broadcast a message  $M$  to all other nodes in the Extended OTIS- $n$ -Cube:

```

Extended OTIS-n-Cube One-to-all Broadcasting
{
  For  $i= 1$  to  $2n$  do
  {
    Every node  $\langle p, g \rangle$  broadcasts  $M$  only to its unvisited
    neighbors
  }
}

```

Using the above gossip type algorithm in OTIS- $n$ -Cube, it only requires  $2n$  units of time which is less than the time needed in the OTIS Cube networks. It is also simpler to apply, in addition to guaranteeing delivering the message from different intermediate nodes.

##### B. Hamiltonian Cycle Structure in the Extended OTIS- $n$ -Cube

This section presents a Hamiltonian cycle structure within the recently proposed interconnection topology. Since Hamiltonian is a cycle in an undirected graph which visits each node exactly once and finally returns to the starting node, the following steps are the description of the proposed algorithm proving that the Extended OTIS- $n$ -Cube topology is Hamiltonian:

1. Let assume that the start node of a path is  $\langle 0, 1 \rangle$ , and  $p = 2$ , where  $p$  represents the bit position of the label.
2. Do  $2^n - 1$  factor moves towards a potential target node by complementing the  $p^{\text{th}}$  bit in the factor label, if the target factor address matches an already visited group or matches the start node group address then increase  $p$

by 1 modulus  $n$ , if all label bits were tested and no move is performed then perform a *nand*; not and; operation between group address and factor address of the current node. The outcome will be the factor address of the node target node.

3. do an optical move from  $\langle g, p \rangle$  to  $\langle p, g \rangle$
4. increase  $p$  by 1 modulus  $n$
5. Repeat steps 2, 3, and 4 as long as the move will not lead to the group label of the start node until the  $2^n - 1$  groups are visited.
6. Finally, construct an optical move back toward the start node.

In the following examples, the dots represent  $n-1$  factor moves of the corresponding nodes; every arrow represents an optical move.

**Example 5:** Hamiltonian cycle within an Extended OTIS-2-Cube topology, Figure 3 shows a representation of such a Hamiltonian cycle.

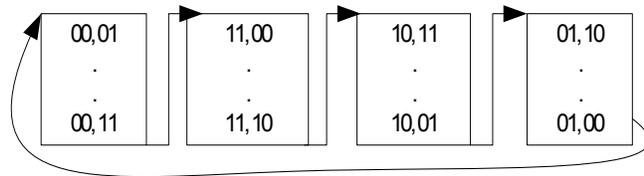


Fig.3 A Hamiltonian cycle in an Extended OTIS-2-Cube.

**Example 6:** Hamiltonian cycle within an Extended OTIS-3-Cube graph, Figure 4 shows a representation of such a Hamiltonian cycle.

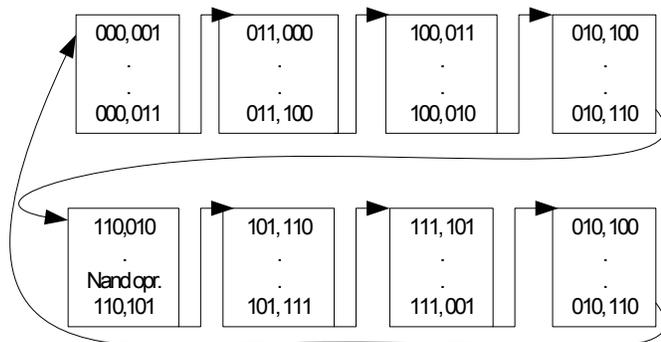
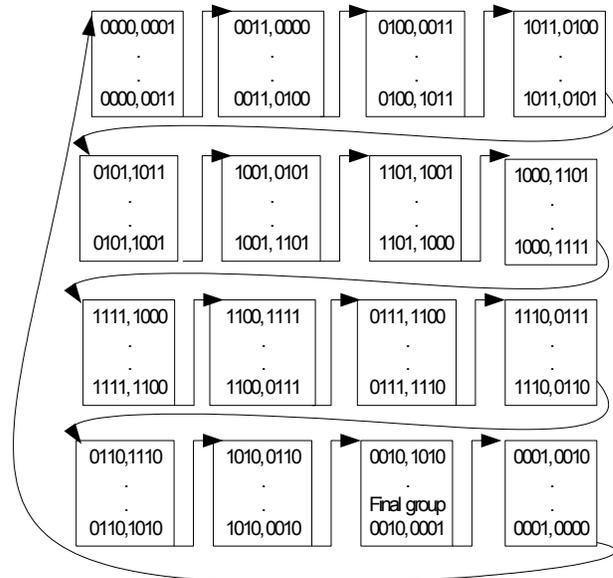


Fig. 4 A Hamiltonian Cycle in an Extended OTIS-3-Cube.

**Example 7:** Hamiltonian cycle within an Extended OTIS-2-Cube graph, Figure 5 shows a representation of such a Hamiltonian cycle.

*C. Minimal Spanning Tree Construction:*

Day [29] proved that an OTIS- $n$ -Cube has a spanning tree. Figure 6 shows a spanning tree broadcast graph in an OTIS-



2-Cube, with starting node  $\langle 00, 00 \rangle$ . The height of a minimal spanning tree for OTIS- $n$ -Cube is always  $2^n + 1$ .

Now, we introduce a spanning tree for the Extended OTIS- $n$ -Cube. Figure 7 shows a spanning tree broadcast graph in an Extended OTIS- $n$ -Cube; where  $n=2$ ; with starting node  $\langle 00, 00 \rangle$ . The height of a minimal spanning tree for Extended OTIS- $n$ -Cube is always  $2^n$ . So the Extended OTIS-2-Cube has a spanning tree height of 4. This means that the height of a minimal spanning tree for Extended OTIS- $n$ -Cube is always less than the height of a minimal panning tree for the OTIS- $n$ -Cube.

Fig.5: A Hamiltonian Cycle in an Extended OTIS-4-Cube.

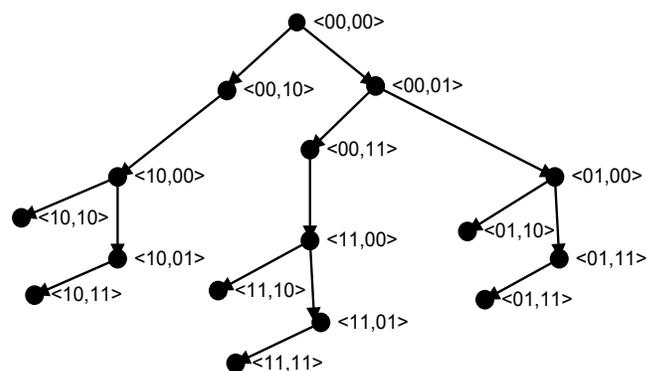


Fig. 6 Minimal spanning tree in an OTIS-2-Cube

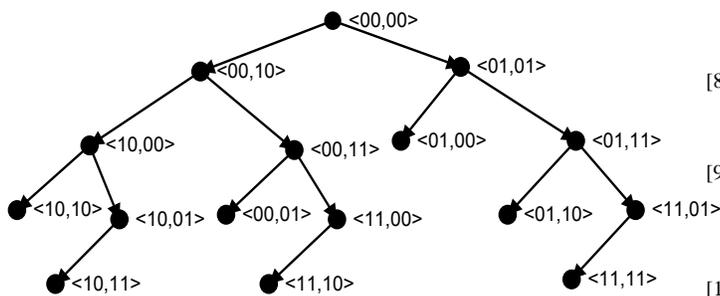


Fig.7: Minimal spanning tree in an Extended OTIS-2-Cube.

A balanced binary tree is commonly defined as a binary tree in which the heights of the two sub trees of every node never differ by more than 1 [32]. It is worth to mention that we can always construct a balanced binary tree for the Extended OTIS- $n$ -Cube, but this is not true for the OTIS- $n$ -Cube. This fact can be verified by referring to figure 6 and figure 7. The balanced binary tree is very important in many mathematical applications especially in searching and optimization implementations [33].

## V. CONCLUSION

This paper introduced the attractive routing algorithm for the Extended OTIS- $n$ -Cube interconnection network. Furthermore, many broadcasting approaches have been presented for this topology. Different types of broadcasting approaches have been presented for this network. An algorithm of forming a Hamiltonian cycle within the Extended OTIS- $n$ -Cube is presented in details showing the good communication properties of the new network. Minimal Spanning Tree and One to all broadcasting approaches are also presented. Examples to show the applicability of these approaches for the Extended OTIS- $n$ -Cube are presented in this paper.

## REFERENCES

[1] T. Tsai, T. Kung, J. Jimmy, J. M. Tan, and L. Hsu, On the Enhanced Hyper-hamiltonian Laceability of Hypercubes, *Proceedings of the 3rd WSEAS International Conference on COMPUTER ENGINEERING and APPLICATIONS (CEA'09)*, pp 62-67.

[2] N-Cube Systems, N-cube Handbook, N-Cube (1986).

[3] J. Rattler, Concurrent Processing: A new direction in scientific computing, Proc. AFIPS Conf. 54 (1985) 157-166.

[4] C.L. Seitz, The cosmic cube, *CACM* 28 (1985) 22-23.

[5] Silicon Graphics, Origin 200 and Origin 2000, Technical Report (1996).

[6] A.M. Awwad, A. Al-Ayyoub, M. Ould-Khaoua, Efficient Routing Algorithms on the OTIS-Networks, *Proceedings of the 3rd International Conference on Information Technology (ACIT'2002)*; The University of Qatar- Doha; Dec. 16-19,138-144.

[7] S. Sahni and C. Wang, BPC Permutations on the OTIS-mesh Optoelectronic Computer, *4th International Conference on*

*Massively Parallel Processing Using Optical Interconnections (MPPOI'97)*, 1997, pp. 130 - 135.

[8] C. Wang and S. Sahni, Basic Operations on the OTIS-mesh Optoelectronic Computer, *IEEE Trans. Parallel and Distributed Systems*, Vol. 9, No. 12, 1998, pp. 1226-1236.

[9] F. Zane, P. Marchand, R. Paturi, and S. Esener, Scalable Network Architecture Using the Optical Transpose Interconnection System (OTIS), *Journal of Parallel and Distributed Computing*, Vol 60, 2000, pp. 521-538.

[10] G. Marsden, P. Marchand, P. Harvey, and S. Esener, Optical Transpose Interconnection System Architecture, *Optics Letters*, Vol. 18, No. 13, 1993, pp. 1083-1085.

[11] H. Ebrahimi-Kahaki, H. Sarbazi-Azad, Broadcast Algorithms on OTIS-Cubes, *Proceedings of Parallel and Distributed Processing with Applications*, 2008, pp. 637 - 642

[12] W. Hendrick , O. Kibar, P. Marchand, C. Fan, D. Blerkom, F. McCormick, I. Cokgor, M. Hansen, and S. Esener, Modeling and Optimisation of the Optical Transpose Interconnection System, *In Optoelectronic Technology Centre, Program Review*, Cornell University, 1995.

[13] B. Mahafzah , R. Tahboub , O. Tahboub, Performance evaluation of broadcast and global combine operations in all-port wormhole-routed OTIS-Mesh interconnection networks, *Cluster Computing*, Vol.13, No.1, 2010, pp. 87-110.

[14] C. Zhao , W. Xiao , B. Parhami, Load-balancing on swapped or OTIS networks, *Journal of Parallel and Distributed Computing*, Vol. 69, No. 4, 2009, pp. 389-399.

[15] A. Krishnamoorthy, P. Marchand, F. Kiamilev, and S. Esener, Grain-size Considerations for Optoelectronic Multistage Interconnection Networks, *Applied Optics*, Vol. 31, No. 26, 1992, pp. 5480- 5507.

[16] D. Benyamina, N. Hallam, A. Hafid On Optimizing the Planning of Multi-hop Wireless Networks using a Multi Objective Evolutionary Approach, *International Journal Of Communications*, Vol 2, No 4, 2008, pp 213-221

[17] M. Popescu, N. Mastorakis, New Aspect on Wireless Communication Networks, *International Journal Of Communications*, Vol 3, No 2, 2009, pp 34-43

[18] V. Heun, E. W. Mayr, Efficient Embeddings into Hypercube-like Topologies, *The Computer Journal*, Vol. 46, No. 6, 2003, pp. 632-644.

[19] R. F. Browne, The Embedding of Meshes and Trees into Degree Four Chordal Ring Networks *The Computer Journal*, Vol. 38, No. 1, 1995, pp. 71-77.

[20] J. Alsadi, A. awwad, A New Fault-Tolerant Routing Algorithm for EOC Interconnection Network, *WSEAS TRANSACTIONS ON COMPUTERS*, Vol. 5, No. 7 , 2006, pp 1474-1480.

[21] H. Hung, J. Fu, G. Chen, Fault-free Hamiltonian cycles in crossed cubes with conditional link faults, *Information Sciences* Vol. 177,2007, pp. 5664–5674.

[22] D. Wang: On Embedding Hamiltonian Cycles in Crossed Cubes, *Parallel and Distributed Systems, IEEE Transactions*, 2008, Vol 19 No. 3, pp. 334 – 346.

[23] I. Zelina , G. Moldovan , I. Tascu, On Embeddings of Hamiltonian Paths and Cycles in Extended Fibonacci Cubes, *American Journal of Applied Sciences*, Vol. 5, No. 11, 2008, pp. 1605-1610.

[24] S. Kao, and P. Wang, "Mutually Independent Hamiltonian Cycles in k-ary n-Cubes when k is Odd", *The American Conference In Applied Mathematics (WSEAS)*, 2010, pp.116-121.

- [25] J. Laudon, D. Lenoski, "System overview of the SGI Origin 200/2000" product line, *EEE Compton '97. Proceedings*, 1997, pp. 150 -156.
- [26] N- Cube Systems, N-cube Handbook, N-Cube (1986).
- [27] J. Rattler, Concurrent Processing: A new direction in scientific computing, *Proc. AFIPS Conference. 54*, 1985, pp. 157-166.
- [28] C.L. Seitz, The cosmic cube, *Communications of the ACM Journal*, Vol. 28, No. 1, 1985, pp. 22-28.
- [29] K.Day and A. Al-Ayyoub, Topological Properties of OTIS-Networks, *IEEE Trans. On Paralle and Distributed systems*, Vol. 13, No. 4, 2002, pp. 359-366.
- [30] J. Al-Sadi, An Extended OTIS-Cube Interconnection Network, *Proceedings of the IADIS International Conference on Applied Computing*, Algarve, Portugal, February 22-25, 2005, Vol. II, pp. 167 – 172.
- [31] V. Kumar, A. Grama, A. Gupta, and G. Karypis, Introduction to Parallel Computing Design and Analysis of Algorithms, The Benjamin/Cummings Publishing Company, Redwood City, CA, 1994.
- [32] K. A. Berman, J. Paul, Algorithms: Parallel, Sequential and Distributed. *Course Technology*, 2005, pp. 113–166.
- [33] S. Sen, R. E. Tarjan, Deletion without rebalancing in balanced binary trees, *Proceeding SODA '10 Proceedings of the Twenty-First Annual ACM-SIAM Symposium on Discrete Algorithms*, 2010

**Jehad A. Al-Sadi** received his B.Sc. degree in Computer Science from Tennessee State University, USA, in 1989, and M.Sc degree in Computer Science from Jackson State University, USA, in 1993. From graduation till 1996.

He was a lecturer in the Department of Computing Science at the Al-Isra University, Jordan. He gained his Ph.D. in Computer Science from the University of Glasgow, U.K. 2002. He joined the Computer Science Department at The Zarka Private University, Jordan from 2002 until 2004. He is currently an associate professor and the Chairman of IT and computing department at Arab Open University, Jordan.

His research interests are: interconnection networks, optical networks, fault-tolerant routing. Also due to his university institutional research interests, he published several papers in e-Learning, and learning management systems (LMS) area.