Extended motion model of autonomous ground vehicle

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***Abstract*—**This article deals with the design and implementation of the mathematical motion model of an autonomous unmanned ground vehicle. This model comes from the specific construction of the vehicle, which was designed in order to verify and demonstrate its autonomous motion possibilities indoors and outdoors. This article presents two different issues of the model: a) computing vehicle's position depending on values of parameters of its motion system and b) deriving values of these parameters according to current demands for movement of the vehicle from a current position to a new position. The second issue is presented in the two versions: basic and extended.

***Keywords*—**Autonomous control, Autonomous motion, Manual control, Model, Unmanned vehicle.

I. INTRODUCTION

THE importance and influence of unmanned vehicles is more and more increasing; there are a lot of possibilities of their utilization both in the civil and military area. Unmanned vehicles can operate in different types of environment; we can distinguish these vehicles as aerial, ground, surface, underwater and space (satellite).

Together with the development of quality of technical equipment and sensorial systems, the software progress and improvement is being carried out. The tendency to automate as many as possible manual actions, which an operator has had to deal with so far, is enormous. According to the degree of automation, we can define unmanned vehicles as semi-autonomous or autonomous.

The main aim is to automate significant processes such as decision support processes, automatic searching, surveying, monitoring and identifying objects and targets, autonomous motion of vehicles in various environments etc. Ground vehicles create real challenges particularly for the last mentioned area; this is a very complex process with many connected steps to be dealt with.

University of Defence in Brno, the Czech Republic, within its long-term research deals with (among others) development of unmanned ground and aerial vehicles determined for tactical utilization. One of many goals of this research consists in solving autonomous motion of ground vehicles indoors and outdoors.

The department of military management and tactics has recently developed an experimental vehicle in order to verify and demonstrate the basic principles of the autonomous motion process. The next parts briefly introduce the design of the motion model applicable also to other types and variants of ground vehicles.

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II. EXPERIMENTAL UNMANNED VEHICLE

The construction of our experimental vehicle is relatively simple. Autonomous motion of this vehicle is based on laser scanning of its surrounding environment. Firstly the configuration of objects and obstructions in the area is created; then it is used for finding an optimal path to a target position. More information about the above mentioned principle can be found in [1], [2].

During the motion of the vehicle along the found path, information about environmental configuration is steadily elaborated and supplied. The whole principle seems to be relatively simple. The main problem, however, consists in keeping the information about the precise position of the vehicle in the area. While moving the vehicle outdoors it can be solved via a GPS receiver integrated with an inertial navigation system elaborating GPS data regularly.

A GPS receiver cannot be used in buildings and closed areas since there is no GPS signal available. Different principles must be used. In our case, it is a set of three activities cooperating together with very precise results:

1. Estimation of an approximated vehicle position via our mathematical model of its motion.
2. Elaboration of the position via a digital magnetometer and accelerometer.
3. Further elaboration of the position using comparison with configurations of the surrounding environment obtained from the initial and new position of the vehicle.

The first step of the whole process, i.e. the design of the mathematical motion model of the vehicle, is elaborated in this article in more detail. The article presents the two basic issues:

1. Estimation of a vehicle position depending on values of parameters of its motion system (so-called manual control).
2. Computation of optimal values of parameters of the motion system, which has to be set in order to move the vehicle into the demanded position (so-called autonomous or position control).

The first model is used for manual control of the vehicle by an operator. The second model is intended for autonomous control when the vehicle moves along the found optimal path step by step. The second model is presented here in the two versions: basic and extended.

III. Model for Manual Control

Before concerning the design of the model itself, we need to define the basic architecture of the vehicle. As already mentioned, the architecture is quite simple. The vehicle is propelled by a pair of servomotors; each servomotor propels one driving wheel. The control is provided via a control unit connected to other peripheral devices such as a communication unit, digital magnetometer and accelerometer, laser scanning device and several other devices (electronic elements, storage batteries, voltage converter etc.).

The architecture of our experimental vehicle is presented in Fig. 1; the size of the chassis is 610 x 500 mm. However, appearance of the vehicle is not the top priority now; the main goal and purpose of the vehicle is to verify and demonstrate the fundamental principles of its autonomous motion.

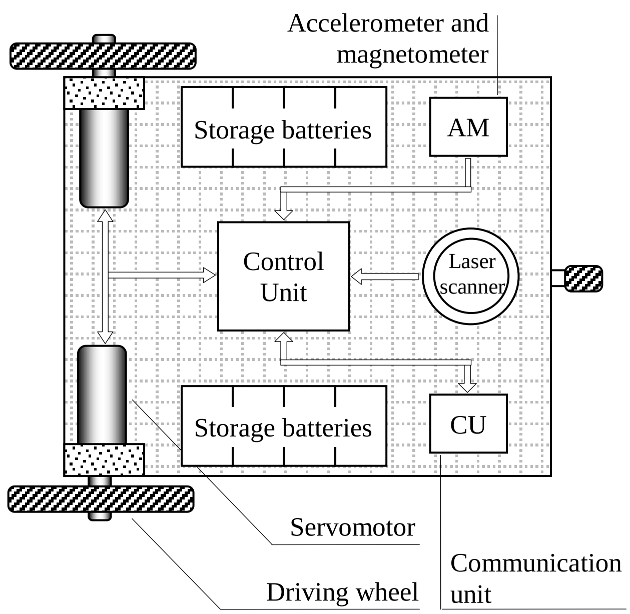


Fig. 1 Architecture of the experimental unmanned vehicle

The model for manual control estimates the position of the vehicle during its motion when controlled by an operator. The operator controls speed and direction of the vehicle by setting rotating speed of both servomotors; i.e. only two variables are set at arbitrary instants of time during manual control of the vehicle.

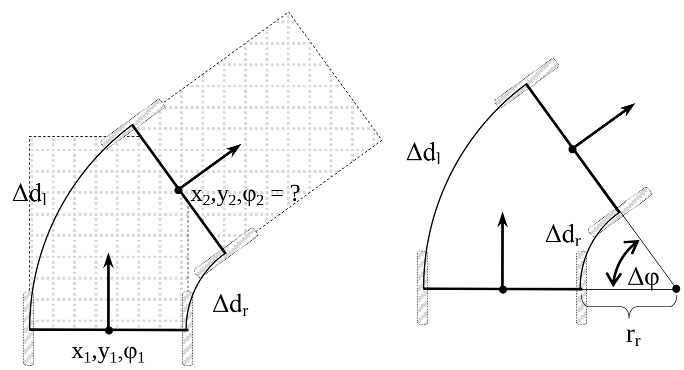
The position of the vehicle during its motion is estimated by reading positions of both servomotors at regular time periods (about 100 ms). Servomotors provide precise values of their actual turning in the real time in the range from 0 to 65,536. Moreover, each servomotor is equipped with a gearbox with the ratio of transmission 1:73; so the rotation of a driving wheel n in the range from 0° to 360° is defined by values from 0 to 73 · 65,536.

A distance covered by a driving wheel during the servomotor rotation within one short time period Δt is expressed in formula (1).

$$\Delta d = \Delta n \cdot \frac{2\pi \cdot r_w}{73 \cdot 65{,}536} \tag{1}$$

where Δd is a distance covered by the driving wheel within a time period Δt [m],
Δn is a change of the servomotor position within a time period Δt [-],
r_w is a radius of the driving wheels [m].

The position of the vehicle in the area is defined by three parameters x, y, φ according to Fig. 2. The aim of this model is to determine the new position of the vehicle depending on the distances Δd_l and Δd_r covered by the left and right driving wheel within a time period Δt (see Fig. 2a). Fig. 2b presents the turning radius of the vehicle r_r during its motion around its right wheel.



a) New vehicle position b) Turning radius

Fig. 2 Position of the vehicle in the area

The first step when computing the new position is to determine the turning radius of the vehicle along one of its driving wheels (see Fig. 2b). The ratio between distances Δd_l and Δd_r can be expressed according to formula (2). Formula (2) can be rewritten as formula (3), which represents the turning radius of the vehicle r_r along its right wheel.

$$\frac{\Delta d_l}{\Delta d_r} = \frac{2\pi \cdot (r_r + w)}{2\pi \cdot r_r} \tag{2}$$

$$r_r = \frac{\Delta d_r \cdot w}{\Delta d_l - \Delta d_r} \quad \text{when} \quad \Delta d_l \neq \Delta d_r \tag{3}$$

where Δd_l and Δd_r are distances covered by the left and right wheels at a time interval Δt [m],
r_r is the turning radius of the vehicle along its right driving wheel [m],
w is the distance between both driving wheels [m].

Formula (4) represents a change of the direction angle $\Delta\varphi$ of the vehicle in its new position.

$$\Delta\varphi = \begin{cases} 0 & \text{when } \Delta d_l = \Delta d_r \\ \dfrac{\Delta d_l}{w} & \text{when } \Delta d_l \neq \Delta d_r \text{ and } r_r = 0 \\ \dfrac{\Delta d_r}{r_r} & \text{when } \Delta d_l \neq \Delta d_r \text{ and } r_r \neq 0 \end{cases} \tag{4}$$

New values of the parameters x_i, y_i, φ_i at a time interval i are represented by formula (5).

$$\varphi_i = \varphi_{i-1} + \Delta\varphi_i$$

$$x_i = \begin{cases} x_{i-1} + \left(\dfrac{\Delta d_l + \Delta d_r}{2}\right) \cdot \sin\varphi_i & \text{when } \Delta d_l = \Delta d_r \\ x_{i-1} + \left(r_r + \dfrac{w}{2}\right) \cdot \left[\sin\left(\varphi_{i-1} + \dfrac{\pi}{2}\right) + \sin\left(\varphi_i - \dfrac{\pi}{2}\right)\right] & \text{when } \Delta d_l \neq \Delta d_r \end{cases} \tag{5}$$

$$y_i = \begin{cases} y_{i-1} + \left(\dfrac{\Delta d_l + \Delta d_r}{2}\right) \cdot \cos\varphi_i & \text{when } \Delta d_l = \Delta d_r \\ y_{i-1} + \left(r_r + \dfrac{w}{2}\right) \cdot \left[\cos\left(\varphi_{i-1} + \dfrac{\pi}{2}\right) + \cos\left(\varphi_i - \dfrac{\pi}{2}\right)\right] & \text{when } \Delta d_l \neq \Delta d_r \end{cases}$$

Fig. 3 presents the simulation of manual control of the vehicle in the area; the figure shows axes of the driving wheels at particular positions of the vehicle and its direction. Table 1 presents values Δd_l and Δd_r at particular steps of the simulation (distance values are in centimeters).

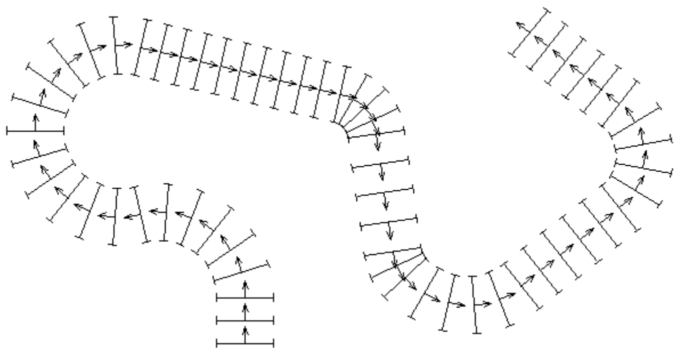


Fig. 3 Simulation of manual control of the vehicle

Table 1 Values Δd_l and Δd_r for the simulation of vehicle's motion according to Fig. 3.

Step *i*	1-2	3-8	9-20	21-30	31-34	35-38
$\boldsymbol{\Delta d_l}$	20	15	30	18	20	26
$\boldsymbol{\Delta d_r}$	20	30	15	18	5	26
Step *i*	39-40	41-45	46-50	51-55	56-60	
$\boldsymbol{\Delta d_l}$	5	15	22	12	20	
$\boldsymbol{\Delta d_r}$	20	30	22	30	20	

IV. Basic Model for Autonomous Control

This model has been developed for autonomous motion of the vehicle along an arbitrary path which can be determined e.g. via the optimal path-finding algorithm [3], [4]. The path is composed of points, which the vehicle passes through step by step from the initial point to the target point.

Generally, this model ensures motion of the vehicle from the current point to the point next in sequence. When the vehicle reaches that point, the whole process is repeated until reaching the last (target) point in sequence.

Input parameters of the model are composed of the vehicle's position x_i and y_i at an interval i and its orientation φ_i as well as the demanded vehicle position at the next step $i+1$ of the solution x_{i+1} and y_{i+1}. As an output there are distances Δd_l and Δd_r which must be covered by the left and right driving wheels during a time interval Δt provided that rotating speed of both wheels is unchanging; speed can be changed only at the beginning of each time period.

Fig. 4 depicts the situation graphically for step $i = 1$. Notice that the demanded direction angle of the vehicle φ_{i+1} is not an input parameter of the model because this cannot be done within one step. This problem is solved in the next chapter dealing with the extended model for autonomous control.

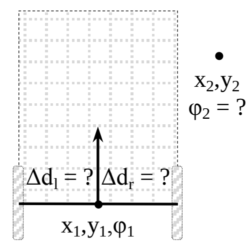
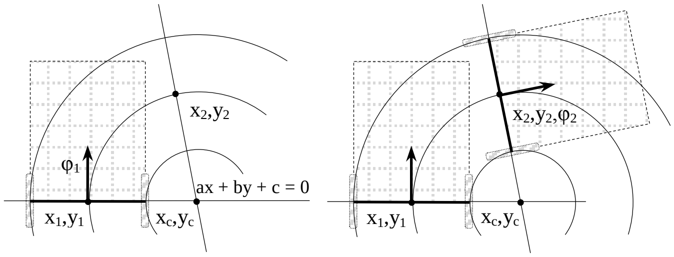


Fig. 4 Input parameters of the model for autonomous control

The problem solution is simple in case that the target point is situated directly on an axis of a direction of the vehicle; formula (6) then expresses distances Δd_l and Δd_r. The sign in the mentioned formula assigns the direction of the wheels rotation and can be determined according to the half-plane given by the axis of driving wheels in which the target point is located. The direction angle is not changed in this case.

$$\Delta d_l = \Delta d_r = \pm\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\Delta\varphi = 0 \tag{6}$$

The situation is more difficult when the target point is not placed in the direction of the vehicle. Fig. 5a and 5b present the solution.



a) Central point b) New position

Fig. 5 Features of the model for autonomous control

Firstly, the central point x_c, y_c should be found; this point means the center of the vehicle rotation and can be found by solution of the equation system (7) with two unknown variables x_c and y_c. The equations were created according to the fact that the distance between the central point and the current and target points is equal and the central point lies on the axis of driving wheels (which is known).

$$(x_1 - x_c)^2 + (y_1 - y_c)^2 = (x_2 - x_c)^2 + (y_2 - y_c)^2$$
$$a \cdot x_c + b \cdot y_c + c = 0 \tag{7}$$

Formula (8) for computing the central point x_c, y_c is found by solving the equation system (7). Analogically, formula (9) can be derived.

$$x_c = \frac{{x_2}^2 + \frac{2 \cdot c \cdot y_2}{b} + {y_2}^2 - {x_1}^2 - \frac{2 \cdot c \cdot y_1}{b} - {y_1}^2}{\frac{2 \cdot a \cdot y_1}{b} - 2 \cdot x_1 - \frac{2 \cdot a \cdot y_2}{b} + 2 \cdot x_2} \quad when \quad b \neq 0$$
$$y_c = -\frac{a \cdot x_c + c}{b} \quad when \quad b \neq 0 \tag{8}$$

$$y_c = \frac{{x_2}^2 + \frac{2 \cdot c \cdot x_2}{a} + {y_2}^2 - {x_1}^2 - \frac{2 \cdot c \cdot x_1}{a} - {y_1}^2}{\frac{2 \cdot b \cdot x_1}{a} - 2 \cdot y_1 - \frac{2 \cdot b \cdot x_2}{a} + 2 \cdot y_2} \quad when \quad a \neq 0$$
$$x_c = -\frac{b \cdot y_c + c}{a} \quad when \quad a \neq 0 \tag{9}$$

When the central point is known we can determine the angle $\Delta\varphi$ according to formula (10) which is based on the equation for computing the mutual angle of two vectors.

$$\Delta\varphi = \operatorname{acos} \frac{(x_c - x_1) \cdot (x_c - x_2) + (y_c - y_1) \cdot (y_c - y_2)}{\sqrt{(x_c - x_1)^2 + (y_c - y_1)^2} \cdot \sqrt{(x_c - x_2)^2 + (y_c - y_2)^2}} \tag{10}$$

Distances Δd_l and Δd_r are defined by formula (11). The sign in the mentioned equations can be determined according to the half-plane given by the axis of driving wheels in which the new positions of the left and right wheels are located.

$$\Delta d_l = \pm\Delta\varphi \cdot \sqrt{(x_c - x_l)^2 + (y_c - y_l)^2}$$
$$\Delta d_r = \pm\Delta\varphi \cdot \sqrt{(x_c - x_r)^2 + (y_c - y_r)^2} \tag{11}$$

where x_l, y_l is the initial position of the left driving wheel,
x_r, y_r is the initial position of the right driving wheel.

Fig. 6 presents the simulation of a vehicle's motion along the predetermined path (the red hatched polygonal line with the yellow points marking demanded positions of the vehicle in a particular step of the simulation). The path contains several sharp breaks; nevertheless the algorithm ensures gradual approaching of the direction of the vehicle towards the curve tangent. Current position of the vehicle is located in its geometric center. Table 2 presents values Δd_l and Δd_r at particular steps during the simulation (distance values are in centimeters).

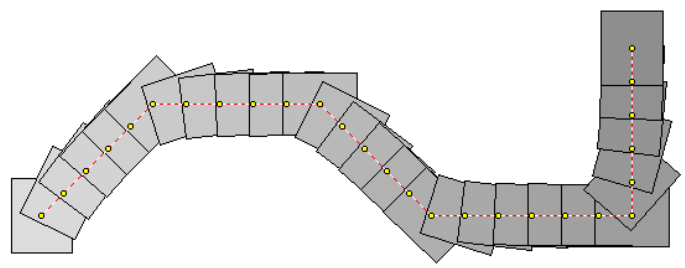


Fig. 6 Simulation of autonomous motion of the vehicle

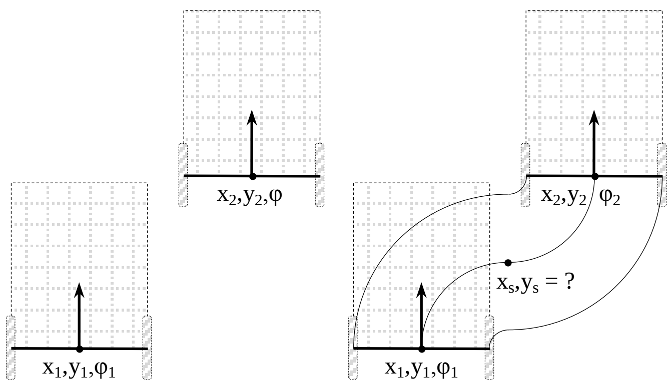
Table 2 Values Δd_l and Δd_r for the simulation according to Fig. 6

Step *i*	1	2	3	4	5	6	7	8	9
Path *x*	10	20	30	40	50	65	80	95	110
Path *y*	10	20	30	40	50	50	50	50	50
$\boldsymbol{\Delta d_l}$	33.1	29.7	27.4	26.3	25.8	36.1	31.3	28.8	27.7
$\boldsymbol{\Delta d_r}$	10.4	19.9	23.3	24.6	25.1	11.2	21.5	25.0	26.3
Step *i*	10	11	12	13	14	15	16	17	18
Path *x*	125	135	145	155	165	175	190	205	220
Path *y*	50	40	30	20	10	0	0	0	0
$\boldsymbol{\Delta d_l}$	27.3	33.1	29.8	27.4	26.3	25.8	11.6	21.7	25.1
$\boldsymbol{\Delta d_r}$	26.7	10.3	19.8	23.3	24.6	25.1	35.1	31.2	28.7
Step *i*	19	20	21	22	23	24	25	26	
Path *x*	235	250	265	265	265	265	265	265	
Path *y*	0	0	0	15	30	45	60	75	
$\boldsymbol{\Delta d_l}$	26.3	27.7	26.9	-9.8	12.8	22.1	25.2	26.3	
$\boldsymbol{\Delta d_r}$	27.7	27.3	27.1	32.5	34.8	30.9	28.6	27.6	

V. Extended model for autonomous control

The extended model for autonomous control widens the basic model mentioned in the previous chapter. This model deals (similarly as the basic model) with vehicle's motion along a predetermined trajectory (path); in addition, there is ensured a requested vehicle's direction at each discrete point of the trajectory (best to its tangent). The big advantage is that we can replace the trajectory by a continual curve (B-spline, Bezier, Hermite, Akima, etc.) along which the vehicle moves smoothly.

Fig. 7a shows the basic parameters of the model; these are the vehicle's position x_i, y_i and its orientation (direction) φ_i in an interval *i* and the requested vehicle's position in the next step x_{i+1}, y_{i+1}, including its direction φ_{i+1} (in the figure the parameters are shown for step $i = 1$). Motion of the vehicle to the new requested position and direction cannot be, however, achieved in one step (in most cases). Therefore, it is necessary to conduct the motion within two independent sub-steps (see Fig. 7b).



a) Parameters

b) Vehicle's motion in two sub-steps

Fig. 7 Parameters and features of the extended model for autonomous control

The output of the extended model is the position x_s, y_s. Taking in account the basic model described in the previous chapter and applying it twice – firstly in the first sub-step to the position x_s, y_s and then in the second sub-step to the position x_{i+1}, y_{i+1}, we are able to move the vehicle to the demanded position x_{i+1}, y_{i+1} and direction φ_{i+1}. Fig. 7b shows the whole principle graphically. The vehicle moves again along a circle within each sub-step.

In most cases, there is not only one point x_s, y_s, through which the vehicle can reach the target point, but there are infinite numbers of them. Parameters and variables needed for finding the point x_s, y_s are shown in Fig. 8.

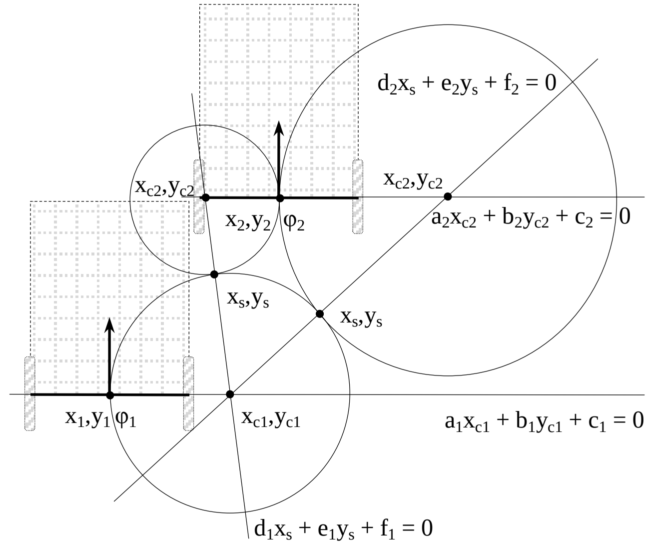


Fig. 8 Parameters and variables for the problem solution

The problem solution is given by the set of 5 equations according to formula (12). Equations contain 6 unknown variables (x_s, y_s, x_{c1}, y_{c1}, x_{c2}, y_{c2}); it results in the possibility of the infinite numbers of different solutions. Moreover, the figure shows that within one solution there are generally two its variants.

$$
\begin{gathered}
a_1 \cdot x_{c1} + b_1 \cdot y_{c1} + c_1 = 0 \\
a_2 \cdot x_{c2} + b_2 \cdot y_{c2} + c_2 = 0 \\
(x_1 - x_{c1})^2 + (y_1 - y_{c1})^2 = (x_s - x_{c1})^2 + (y_s - y_{c1})^2 \\
(x_2 - x_{c2})^2 + (y_2 - y_{c2})^2 = (x_s - x_{c2})^2 + (y_s - y_{c2})^2 \\
\sqrt{(x_{c2} - x_{c1})^2 + (y_{c2} - y_{c1})^2} = \sqrt{(x_s - x_{c1})^2 + (y_s - y_{c1})^2} + \sqrt{(x_s - x_{c2})^2 + (y_s - y_{c2})^2}
\end{gathered}
\tag{12}
$$

When solving the problem, we need to eradicate one unknown variable from the set of equations. One of possible solutions is to set the variable x_{c1} (y_{c1} respectively) randomly and then to compute the variable y_{c1} (x_{c1} respectively) from the first equation of formula (12). The best way is to repeat the mentioned procedure for several times (for various values of x_{c1} or y_{c1}) and then to select the best solution. Another possibility is to determine the point x_{c1}, y_{c1} via some heuristic or any other method.

When we know the point x_{c1}, y_{c1}, we can compile formula (13) containing two equations with two unknown variables (x_{c2}, y_{c2}).

$$
\begin{gathered}
a_2 \cdot x_{c2} + b_2 \cdot y_{c2} + c_2 = 0 \\
\sqrt{(x_{c2} - x_{c1})^2 + (y_{c2} - y_{c1})^2} = \sqrt{(x_1 - x_{c1})^2 + (y_1 - y_{c1})^2} + \sqrt{(x_2 - x_{c2})^2 + (y_2 - y_{c2})^2}
\end{gathered}
\tag{13}
$$

The solution of introduced equations is relatively complex; it leads to the form of a quadratic equation (14) where k_1, k_2, k_3 are constants computing according to formulas (15), (16) and (17). Mentioned formulas are valid only when $b_2 \neq 0$. Analogically we can also assemble the formulas when $a_2 \neq 0$.

$$k_1 \cdot {x_{c2}}^2 + k_2 \cdot x_{c2} + k_3 = 0 \tag{14}$$

$$\begin{aligned} k_1 = {} & 4 \cdot (x_2 - x_{c1})^2 - 4 \cdot (x_1 - x_{c1})^2 - 4 \cdot (y_1 - y_{c1})^2 \\ & +4 \cdot \frac{{a_2}^2}{{b_2}^2} \cdot ((y_2 - y_{c1})^2 - (x_1 - x_{c1})^2 - (y_1 - y_{c1})^2) - 8 \cdot \frac{a_2}{b_2} \cdot (x_2 - x_{c1}) \cdot (y_2 - y_{c1}) \end{aligned} \tag{15}$$

$$\begin{aligned} k_2 = {} & 8 \cdot \frac{a_2 c_2}{{b_2}^2} \cdot ((y_2 - y_{c1})^2 - (x_1 - x_{c1})^2 - (y_1 - y_{c1})^2) - 8 \cdot \frac{c_2}{b_2} \cdot \left((x_2 - x_{c1}) \cdot (y_2 - y_{c1})\right) \\ & -4 \cdot \frac{a_2}{b_2} \cdot \left((y_2 - y_{c1}) \cdot ({x_{c1}}^2 - {x_2}^2 + {y_{c1}}^2 - {y_2}^2 - (x_1 - x_{c1})^2 - (y_1 - y_{c1})^2)\right) \\ & -8 \cdot y_2 \cdot \frac{a_2}{b_2} \cdot ((x_1 - x_{c1})^2 + (y_1 - y_{c1})^2) + 8 \cdot x_2 \cdot ((x_1 - x_{c1})^2 + (y_1 - y_{c1})^2) \\ & +4 \cdot (x_2 - x_{c1}) \cdot ({x_{c1}}^2 - {x_2}^2 + {y_{c1}}^2 - {y_2}^2 - (x_1 - x_{c1})^2 - (y_1 - y_{c1})^2) \end{aligned} \tag{16}$$

$$\begin{aligned} k_3 = {} & 4 \cdot \frac{{c_2}^2}{{b_2}^2} \cdot ((y_2 - y_{c1})^2 - (x_1 - x_{c1})^2 - (y_1 - y_{c1})^2) \\ & -4 \cdot \frac{c_2}{b_2} \cdot \left((y_2 - y_{c1}) \cdot ({x_{c1}}^2 - {x_2}^2 + {y_{c1}}^2 - {y_2}^2 - (x_1 - x_{c1})^2 - (y_1 - y_{c1})^2)\right) \\ & -8 \cdot y_2 \cdot \frac{c_2}{b_2} \cdot ((x_1 - x_{c1})^2 + (y_1 - y_{c1})^2) - 4 \cdot ({x_2}^2 + {y_2}^2) \cdot ((x_1 - x_{c1})^2 + (y_1 - y_{c1})^2) \\ & +({x_{c1}}^2 - {x_2}^2 + {y_{c1}}^2 - {y_2}^2 - (x_1 - x_{c1})^2 - (y_1 - y_{c1})^2)^2 \end{aligned} \tag{17}$$

The solution of the quadratic equation (14) is given by formula (18). There is no solution when ${k_2}^2 - 4 \cdot k_1 \cdot k_3 < 0$, only one solution when $k_1 = 0$ or ${k_2}^2 - 4 \cdot k_1 \cdot k_3 = 0$, otherwise two solutions (see the example in Fig. 8). When there is at least one solution, the value y_{c2} can be found from the first equation (13) according to formula (19).

$$x_{c2_{1,2}} = \frac{-k_2 \pm \sqrt{{k_2}^2 - 4 \cdot k_1 \cdot k_3}}{2 \cdot k_1} \tag{18}$$

$$y_{c2_{1,2}} = -\frac{a_2 \cdot x_{c2_{1,2}} + c_2}{b_2} \quad when \quad b_2 \neq 0 \tag{19}$$

When knowing values x_{c2}, y_{c2}, we can assemble (on the basis of Fig. 8) equations (20) from which we can compute the requested point (points) $x_{s_{1,2}}$, $y_{s_{1,2}}$. Its solution is not introduced here because of the limited range of this article.

$$\begin{gathered} d_{1,2} \cdot x_{s_{1,2}} + e_{1,2} \cdot y_{s_{1,2}} + f_{1,2} = 0 \\ \left(x_{s_{1,2}} - x_{c1}\right)^2 + \left(y_{s_{1,2}} - x_{c1}\right)^2 = (x_{c1} - x_1)^2 + (y_{c1} - y_1)^2 \\ \left(x_{s_{1,2}} - x_{c2_{1,2}}\right)^2 + \left(y_{s_{1,2}} - x_{c2_{1,2}}\right)^2 \\ = \left(x_{c2_{1,2}} - x_2\right)^2 + \left(y_{c2_{1,2}} - y_2\right)^2 \end{gathered} \tag{20}$$

In Fig. 9 there are shown four possibilities of vehicle's motion which can be found within one solution. The choice of one of the possibilities depends on the initial vehicle's direction and the requested direction at the target point. It is always necessary to select the right version so that the result corresponds with our requirements.

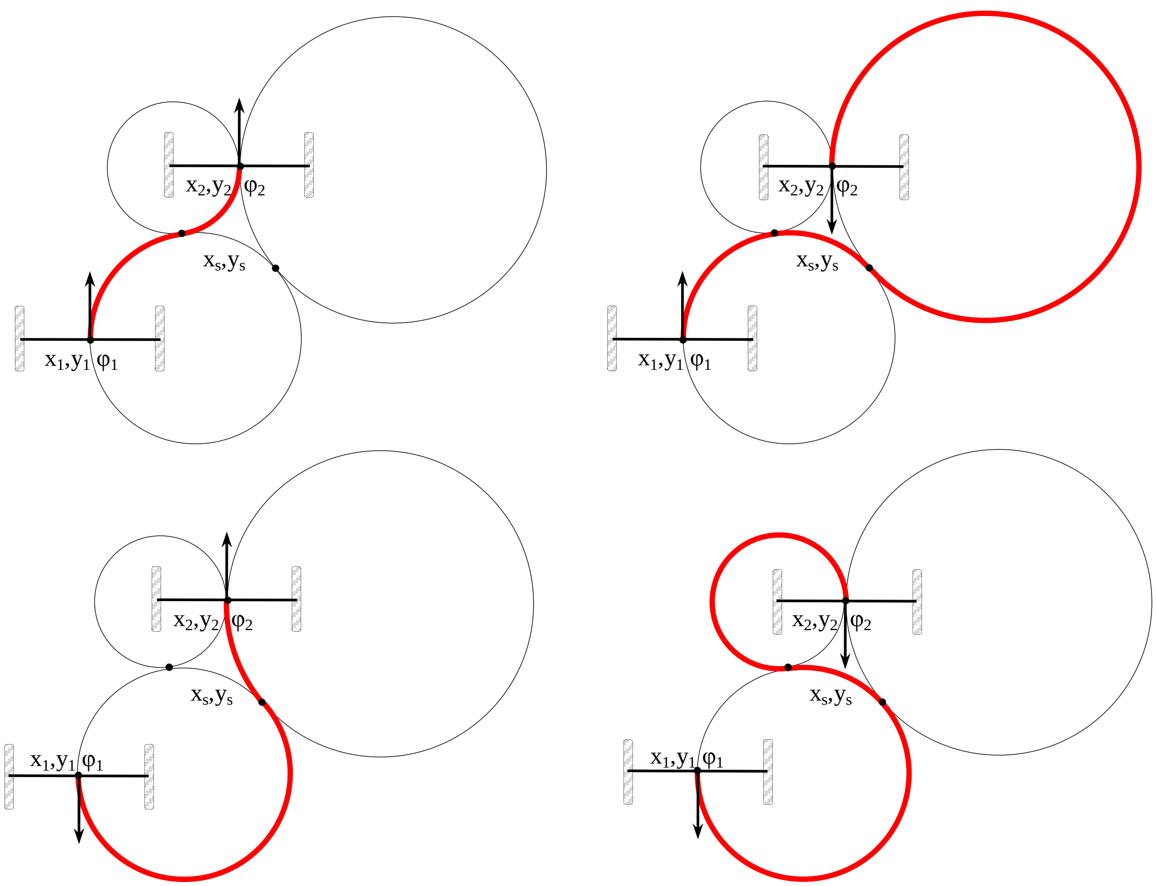


Fig. 9 Basic possibilities of vehicle's motion given by the mathematical solution

Fig. 10 shows next three examples with three different input values. Examples have been solved on the model implemented into our simulator. Table 3 contains values of important input and output variables for each example (length units are expressed in millimeters).

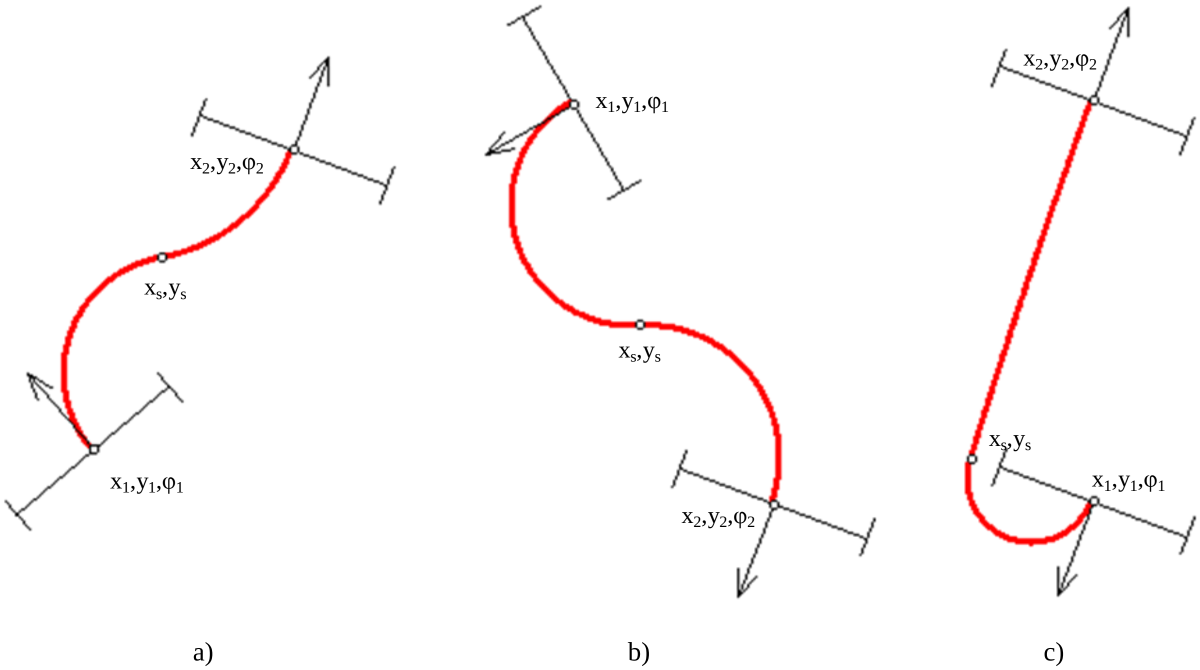


Fig. 10 Three examples of the problem solution

Table 3 Values of input and output variables according to Fig. 10

Example a)				Example b)				Example c)			
x_1	0.0	x_{c1}	45.0	x_1	0.0	x_{c1}	30.0	x_1	0.0	x_{c1}	-30.0
y_1	0.0	y_{c1}	37.8	y_1	0.0	y_{c1}	-52.0	y_1	0.0	y_{c1}	10.9
φ_1	320°	x_{c2}	20.6	φ_1	240°	x_{c2}	38.2	φ_1	200.0	x_{c2}	3696
x_2	100.0	y_{c2}	178.9	x_2	100.0	y_{c2}	-177.5	x_2	0.0	y_{c2}	-1138
y_2	150.0	Δd_{l1}	228.1	y_2	-200.0	Δd_{l1}	26.8	y_2	200.0	Δd_{l1}	253.4
φ_2	20°	Δd_{r1}	18.3	φ_2	200°	Δd_{r1}	295.1	φ_2	20.0	Δd_{r1}	-55.9
x_s	35.0	Δd_{l2}	36.2	x_s	33.9	Δd_{l2}	229.8	x_s	-60.5	Δd_{l2}	192.0
y_s	96.5	Δd_{r2}	141.3	y_s	-111.8	Δd_{r2}	31.4	y_s	20.4	Δd_{r2}	187.1

VI. Conclusion

Autonomous motion of unmanned vehicles is the very popular and complex topic being solved at many universities and both civil and military research institutes. We can find a lot of different models and approaches to this contemporary issue. Some of them are mentioned in [6]-[16].

The article presents the basic design for autonomous motion of the unmanned ground vehicle developed at the University of Defence in Brno for experimental purposes. The vehicle demonstrates the algorithm of autonomous motion; this topic is long-term dealt with at the Department of military management and tactics.

In the future, we are going to enhance and verify the algorithm thoroughly. The first step will consist in extending the scanning process of the surrounding environment into the third dimension. The general objective is to apply our algorithm to the autonomous unmanned ground vehicle [2], which is being developed at the University of Defence; this vehicle is used especially for reconnaissance and combat purposes. However, the mathematical model will have to be adjusted to the different physical parameters and features of the vehicle.

This paper extends the basic model for autonomous control [5]; it is possible to move the vehicle to the new position following the condition of the demanded vehicle's direction.

The predetermined path can be replaced with a continuous curve (B-spline, Bezier, Hermite, Akima, polynomial, etc.); the vehicle copies the tangent of the curve at each point.

However, both theoretical experiments on our simulator and practical tests in the real terrain show that the basic model described in the chapter IV. suffice for autonomous motion of the vehicle along a path. As shown in Fig. 6, the vehicle copies the tangent of the path. So the extended model in the chapter V. only widens this complex topic.

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