A new proposal for computing portfolio value-at-risk for semi-nonparametric distributions

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***Abstract*—**This paper proposes a semi-nonparametric (SNP) methodology for computing portfolio value-at-risk (VaR) that is more accurate than both the traditional Gaussian-assumption-based methods implemented in the software packages used by risk analysts (RiskMetrics), and alternative heavy-tailed distributions that seem to be very rigid to incorporate jumps and asymmetries in the distribution tails (e.g. the Student's t). The outperformance of the SNP distributions lies in the fact that Edgeworth and Gram-Charlier series represent a valid asymptotic approximation of any "regular" probability density function. In fact these expansions involve general and flexible parametric representations capable of featuring the salient empirical regularities of financial data. Furthermore these distributions can be extended to a multivariate context and may be estimated in several steps and thus we propose to estimate portfolio VaR in three steps: Firstly, estimating the conditional variance and covariance matrix of the portfolio consistently with the multivariate SNP distribution; Secondly, estimating the univariate distribution of the portfolio constrained to the portfolio variance obtained from the previous step; Thirdly, computing the corresponding quantile of the portfolio distribution by implementing straightforward recursive algorithms. We estimate the VaRs obtained with such methodology for different bivariate portfolios of stock indices and interests rates finding a clear underestimation (overestimation) of VaR measures obtained from the traditional Gaussian- (Student's t-) based methods compared to our SNP approach.

***Keywords*—** Edgeworth and Gram-Charlier series, GARCH models, multivariate densities, semi-nonparametric distributions, Value-at-Risk.

I. INTRODUCTION

Recent stock market crashes have shown that the traditional methods have failed in providing accurate value-at-risk (VaR hereafter) measures. In fact most risk management software packages (e.g. RiskMetrics) implement VaR methodologies assuming a Gaussian distribution of returns that, although simplifies the VaR calculation (only requiring the estimation of the portfolio variance and covariance matrix) is not reliable. For this reason different distributions have been proposed to account for the heavy tails and asymmetries featured by high frequency data, the Student's t being the most widespread – see [1] for a recent application. Nevertheless the parametric approaches may also be insufficient to accurately capture risk since most of them depend on a few parameters to account for the shape of the true distribution.

On the other hand, the semi-nonparametric (SNP hereafter) methods overcome all these shortcomings through a flexible specification admitting as many parameters as necessary to approximate the underlying (true) distribution of asset returns series. Particularly it is well-known that any frequency distribution can be rewritten in terms of an infinite expansion of Edgeworth and Gram-Charlier series – see [2] for the early applications of these series in econometrics. The empirical applications, however, require the truncation of the expansions, which may affect the positiveness of the density but not the quality of the approximation and the risk measures based on it ([3] introduced a simple transformation to solve this problem and [4] analyzed the resulting "positive" SNP in detail).

Furthermore the Gram-Charlier family of densities has been generalized to a multivariate framework and applied for financial purposes in [5], [6] and [7]. These articles define a whole family of multivariate SNP distributions that not only preserve the nice properties of their univariate counterparts (i.e. generality, flexibility and simplicity) but also satisfy other interesting properties: (i) Their marginal (univariate) distributions behave within the same SNP family; (ii) they are invariant from linear transformations; (iii) they can be consistently estimated in two and three steps (i.e. conditional means and variances can be estimated previously and independently from the rest of the distribution parameters); (iv) their statistical properties, including their cumulative distribution function (cdf), and thus their quantiles, can be easily derived.

All these properties allow establishing a very straightforward procedure to calculate portfolio VaR for SNP distributions that may me applied even for large portfolios. Such a procedure may be divided into three steps. Firstly, the time-varying conditional variance and covariance matrix of the portfolio is estimated consistently with the SNP hypothesis. It must be noted that for large portfolios this step can be simplified by estimating portfolio means and variances in the marginal univariate distributions – in this case consistency is achieved even if a Gaussian distribution is assumed instead of a SNP density since the log-likelihood decomposition proposed in [8] is feasible – and estimating the rest of the parameters in the multivariate density evaluated in the standardized (zero mean and unit variance) variables.

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Secondly, portfolio distribution parameters are estimated in the univariate SNP density constrained to the portfolio mean and variance obtained from the estimates of the previous step. Thirdly, the appropriate quantile of the estimated portfolio distribution given a probability (i.e portfolio VaR) is obtained by implementing recursive algorithms.

The next Section (2) describes the methodology for calculating a portfolio's VaR under both the Normal and the SNP distributions in detail, and Section 3 displays the empirical results obtained for a bivariate portfolio of S&P500 index and long run interest rates in United States with different weights and specifications. These results show that traditional methods provide underestimated VaR measures (especially for high confidence levels) but heavy-tailed distributions overestimate portfolio VaR compared to the more general and flexible SNP approach. The conclusions of the paper are gathered in Section 4.

II. PORTFOLIO VAR FOR GAUSSIAN AND SNP DISTRIBUTIONS

In this section we summarize the statistical procedures to compute portfolio VaR under either the traditional Gaussian assumption or the more general SNP specification. For this purpose we model the joint behavior of a group of asset returns as a random vector,

$$x_t' = (x_{1t}, x_{2t}, \cdots, x_{nt}) \in \Re^n, \tag{1}$$

whose conditional distribution, denoted by

$$X_t \big| \Omega_{t-1} \sim F_t(\mu_t(\phi), \Sigma_t(\alpha), \delta), \tag{2}$$

is parameterized in terms of its first (μ_t) an second (Σ_t) conditional moments (depending on the parameter vectors ϕ and α, respectively) and the shaping parameters, included in vector δ that, for the sake of simplicity, is considered constant over time. Note that Ω_{t-1} stands for the available information set at time *t*.

Without loss of generality we model conditional means and variances by AR(1) and GARCH(1,1) processes, respectively, as shown in equations (3) and (4). The former model has been proved to capture the small predictable component of conditional mean of financial returns and the second incorporates the salient features of conditional variances of asset returns, i.e. volatility clustering or persistence – see the seminal papers in [9] and [10] or, more recently, in [11].

$$\mu_{it} = \phi_{0i} + \phi_{1i} x_{it-1} \tag{3}$$

$$\sigma_{it}^2 = \alpha_{0i} + \alpha_{1i}\sigma_{i,t-1}^2 + \alpha_{2i}\varepsilon_{i,t-1}^2 \tag{4}$$

where $\varepsilon_{it} = x_{it} - \mu_{it}$, $|\phi_{1i}| < 1$, $\alpha_{0i} > 0$, $0 \le \alpha_{si} \le 1$ $\forall s = 1,2$ and $\alpha_{1i} + \alpha_{2i} < 1$, $\forall i = 1,2,...,n$. Alternatively, many other models to capture asymmetries (leverage effect) can be used as in [12] or [13]. Moreover, we also assume constant correlation coefficients (ρ_{ij}) (i.e. the CCC model), as proposed in [14], although the extension to the DCC model by [8] is also possible as shown in [7].

Given a vector,

$$\theta' = (\theta_1, \theta_2, \cdots, \theta_n) \in \Re^n, \tag{5}$$

such that $0 \le \theta_i \le 1$ $\forall i = 1,2,...,n$ and $\sum_{i=1}^{n} \theta_i = 1$, a portfolio can be defined by the linear convex combination in equation (6).

$$y_t = \theta' x_t = \sum_{i=1}^{n} \theta_i x_{it} \tag{6}$$

It is clear that the conditional mean and variance of the portfolio is given by

$$\mu_{yt} = \theta' \mu_t = \sum_{i=1}^{n} \theta_i \mu_{it} \tag{7}$$

$$\sigma_{yt}^2 = \theta' \Sigma_t \theta = \sum_{i=1}^{n} \theta_i^2 \sigma_{it}^2 + \sum_{i=1}^{n} \sum_{j=1, j \ne i}^{n} \theta_i \theta_j \rho_{ij} \sigma_{it} \sigma_{it} \tag{8}$$

The portfolio VaR is the maximum expected loss of the portfolio with a given probability (*p*) and over a certain time horizon – see e.g. [15]. Hence the portfolio VaR at time *t* and with the probability *p,* denoted by VaR(*p*) is the corresponding quantlie of the probability density function (pdf hereafter) of the portfolio at this time, $f(\bullet)$.

$$p = \int_{-\infty}^{-VaR(p)} f_y(y_t / \phi, \alpha, \delta, \theta) dy_t \tag{9}$$

For example, if we assume that the vector x_t is Normally distributed, the VaR computation is straightforward, since this distribution depends only on its first and second moments and this distribution is invariant with respect to linear transformations. Therefore, in terms of the quantile of the N(0,1) for a the confidence level *p*, $\lambda(p)$, the portfolio VaR can be obtained as

$$VaR(p) = \mu_{yt} - \sigma_{yt}\lambda(p). \tag{10}$$

In other words, if financial returns are Normally distributed, the portfolio VaR will be directly obtained through the estimates of the parameters for the location and scale models in equations (7) and (8) and the multivariate Gaussian pdf - equation (11) – quantiles.

$$G(x_t) = (2\pi)^{-N/2}\left|\Sigma_t^{-1/2}\right| \exp\left\{-\frac{1}{2}(x_t-\mu_t)'\Sigma_t^{-1}(x_t-\mu_t)\right\} \tag{11}$$

Unfortunately, asset returns are not Normal and, then, VaR measures obtained through this method are clearly misleading. In fact the heavier the tails of the portfolio distributions are and the more skewed the distribution is, the bigger the underestimation of risk is obtained by applying such procedure.

In order to tackle this problem in this paper we propose the use of more flexible SNP density specifications based on Gram-Charlier series. The rationale of the use of these series is based on the known fact that any frequency function, $h(x)$, can be expanded in a (infinite) series of derivatives of the standard – i.e. N(0,1) – Gaussian density, $g(\bullet)$, as shown in equation (12),

$$h(x) = \left[1+\sum_{j=1}^{\infty} k_j H_j(x)\right] g(x) \tag{12}$$

where $H_s(\bullet)$ stands for the so-called Hermite polynomials that can be obtained through the following derivatives

$$\frac{d^j g(x)}{dx^j} = (-1)^j g(x) H_j(x). \tag{13}$$

The Hermite polynomials hold interesting properties; among which the orthogonality – see equation (14) – is the basis of the up to one integration of SNP densities, which are based on them – see [16] for further details about these polynomials.

$$\int H_i(x)H_j(x)g(x)dx = \begin{cases} 0 & if \quad i \neq j \\ i! & if \quad i = j! \end{cases} \tag{14}$$

For empirical purposes the Gram-Charlier expansion needs to be truncated at some degree m. Therefore the SNP distribution of variable x_{it} can be described as

$$f_i(x_{it}) = \frac{1}{\sigma_{it}}\left[1+\sum_{j=2}^{m} \delta_{ji} H_j(z_{it})\right] g(z_{it}) \tag{15}$$

where

$$z_{it} = \frac{x_{it}-\mu_{it}}{\sigma_{it}}, \tag{16}$$

The truncated series, however, do not guarantee positiveness. This problem has been tackled from different perspectives; squaring the expansions (and scaling the resulting density) – as proposed in [3] – and taking into account the positivity regions in terms of skewness and kurtosis – as in [17] – being the most fruitful alternatives. However, if maximum likelihood procedures converge they necessarily do it to estimates that guarantee positiveness – as argued in [18]. Furthermore, these authors show that the densities based on this type of expansions feature the salient empirical regularities of financial returns such as thick tails or asymmetries.

The good performance of the Gram-Charlier densities has been recently extended to the multivariate framework in [5] and [6], which provided a general family of multivariate distributions encompassing most of the univariate SNP alternatives proposed in finance to account for the asset returns distribution. The simplest case is given in equation (17),

$$F(x_t) = G(x_t) + \left[\prod_{i=1}^{n} \frac{1}{\sigma_{it}} g(z_{it})\right]\left[\sum_{i=1}^{n} q_i(z_{it})\right] \tag{17}$$

where $G(\bullet)$ represents the multivariate Normal pdf in equation (11), $g(\bullet)$ stands for the N(0,1) pdf, z_{it} are the standardized variables in equation (16) and

$$q_i(\bullet) = \sum_{j=2}^{m} \delta_{ji} H_j(\bullet) \tag{18}$$

are the corresponding linear combination of Hermite polynomials (without loss of generality we assume the same truncation order, m, for every dimension).

It is noteworthy that the Gaussian distribution is nested in the family of Gram-Charlier distributions (it can be trivially obtained by constraining $\delta_{ji} = 0$, $\forall i = 1,2,...,n$ and $\forall j = 1,2,...,m$). Even more, the portfolio distribution can be obtained in terms of the moments (or the parameters) of the multivariate joint distribution. For example, if we consider that portfolio distribution can be approximated by the Gram-Charlier distribution expanded to the fourth term and considering that $\delta_{ji} = 0 \quad \forall j \neq 4$ and $\forall i = 1,2,...,n$, the density can be expressed either as a function of δ_4 – see equation (19) – or the fourth moment of the portfolio variable as in equation (20).

$$f(y_t) = \frac{1}{\sigma_{yt}} g(\eta_t)\left\{1+\delta_{4y}(\eta_t^4 - 6\eta_t^2 + 3)\right\} \tag{19}$$

$$f(y_t) = \frac{1}{\sigma_{yt}} g(\eta_t)\left\{1+\frac{E\left[y_t^4\right]-3}{24}(\eta_t^4 - 6\eta_t^2 + 3)\right\} \tag{20}$$

Note that in equations (19) and (20) it follows that

$$\eta_t = \frac{y_t - \mu_{yt}}{\sigma_{yt}}. \quad (21)$$

Then, portfolio distributions can be obtained in terms of moments an co-moments of the marginal densities, e.g. for a simple portfolio of two assets it holds that

$$E\left[y_t^4\right] = w_1^4 E\left[x_{1t}^4\right] + w_2^4 E\left[x_{2t}^4\right] + 6 w_1^2 w_2^2 E\left[x_{2t}^2 x_{2t}^2\right]. \quad (22)$$

Note that the co-volatility, i.e. $E\left[x_{1t}^2 x_{2t}^2\right]$, among both assets can be obtained from the moments of the Gaussian density – see [6] for more details about cross-moments of multivariate Gram-Charlier densities. Alternatively, the univariate portfolio distribution can be directly estimated subject to the portfolio mean and variance – previously estimated from equations (7) and (8) under the corresponding multivariate SNP distribution. In the empirical application presented in next section we adopt this latter approach.

Finally, once the portfolio distribution has been estimated its quantiles for the given probability p must be obtained to compute VaR as in equation (10). For this purpose it must be taken into account that for the standard (i.e. zero mean and unitary variance) Gram-Charlier pdf distribution,

$$p = \int_{-\infty}^{-\lambda(p)^*} g(r)dr - g(\lambda(p)^*) \sum_{j=2}^{m} \delta_j H_{j-1}(\lambda(p)^*). \quad (23)$$

Note that the quantiles, $\lambda(p)^*$, for the SNP Gram-Charlier densities can be obtained through their N(0,1) counterparts. Therefore, it is clear that for extreme values the quantiles are bigger (i.e. the distribution has fatter tails than the Normal). This is the reason behind the fact that the SNP methods provide more accurate VaR measures. It is not only the more flexibility of the distribution to adapt different shapes with its general parametric approximation (more flexible than other parametric alternatives proposed in financial literature) but also the fact that if the expansion is large enough we can consider that it accurately approximates the true portfolio distribution.

III. EMPIRICAL APPLICATION

The model shown in previous section was estimated by maximum likelihood (ML hereafter) for two-asset portfolios. We used daily data of continuously compounded returns, measured as

$$y_t = 100 \log(P_t / P_{t-1}), \quad (24)$$

where P_t stands for the corresponding asset prices at time t. The particular empirical example shown in this section considers portfolios of a stock index (Dow Jones industrials index) and the long run interest rates in United States (10 year benchmark bond yields from US Treasury), using different weights (either θ_1=0.1, θ_1=0.5 or θ_1= 0.9, being θ_1 the weight for the first portfolio asset). The results, however, can be replicated using most financial series of high frequency data. The sample period used for both series ranges from 4/1/93 to 28/5/06 (3495 observations).

We apply the methodology explained in previous section assuming either Normal, SNP or Student's t distribution and also accounting for conditional heteroskedasticity. Despite the fact that the resulting models are highly non-linear, the implementation of ML algorithms does not seem to be very demanding. However, the accurate selection of the initial values for the density (d_{2i}, d_{4i}, d_{6i} and d_{8i}, $\forall i$=1,2), the AR(1) (ϕ_{0i} and ϕ_{1i}, $\forall i$=1,2) and the GARCH (α_{0i}, α_{1i} and α_{2i},$\forall i$=1,2) parameters may help the algorithms to achieve a rapid convergence. The choice of initial values is based on the direct relation among the density parameters and the density moments. For example, d_{2i} captures part of the distribution variance, whilst d_{4i} accounts for the excess kurtosis. Moreover, the asymmetries depend on the odd parameters of the distribution, which were removed after having tested that they were not significantly different from zero (see [18] for a description of the distribution moments). The expansions were also truncated at the 8th order according to accuracy criteria.

Table I displays parameter estimates of the joint distribution of portfolio variables under the Normal (NOR), and SNP distribution, as well as those for the multivariate Student's t (ST) with v degrees of freedom – see [19] for the details on this density. The estimates confirm the common pattern of high frequency financial data: unpredictable conditional means (ϕ_{1i}, $\forall i$=1,2, is not significant), high persistence in conditional variances ($\phi_{0i}+\phi_{1i}$, $\forall i$=1,2, is very close to one) and leptokurtosis ($d_{4i} > 0$, $\forall i$=1,2, and v is quite low). Furthermore there are also strong evidence of extreme values since expansions up to the eighth term are required (d_{6i} and d_{8i}, $\forall i$=1,2, are clearly significant). Finally, the correlation among both assets is high and negative, as expected.

The log-likelihood values also show clear evidence in favor the multivariate SNP distribution. It must be also noted that the straightforward implementation of the LR test[1] induces a strong rejection of the Normal density compared to the SNP, since the multivariate Normal is nested on the SNP. In consequence, the portfolio's VaR under normal assumptions is clearly misleading for asset returns. This evidence has previously been stated by different authors, e.g. [21] or [22] compare Normal-VaRs to those obtained under a Student's t distribution or a semi-parametric method.

[1] LR=2[lnL(SNP)–lnL(Normal)]~χ^2_q, q being the number of SNP distribution parameters. Note that this test is more general than other tests such as the traditional normality test by [20], because it accounts not only for the third and fourth moments (captured by d_{3i} and d_{4i} respectively) but also for other moments (incorporated in the other parameters of the distribution).

TABLE I

BIVARIATE DISTRIBUTION ESTIMATES

Param.[a]	NOR	SNP	ST
ϕ_{01}	$.48\times10^{-3}$	$.48\times10^{-3}$	$.48\times10^{-3}$
ϕ_{11}	.016*	.016*	.016*
α_{01}	$.70\times10^{-6}$	$.73\times10^{-6}$	$.44\times10^{-6}$
α_{11}	.965	.967	.966
α_{21}	.036	.037	.020
δ_{21}		−.069*	
δ_{41}		.092	
δ_{61}		.010	
δ_{81}		$.194\times10^{-2}$	
ϕ_{02}	$.12\times10^{-3}$*	$.12\times10^{-3}$*	$.12\times10^{-3}$*
ϕ_{12}	.021*	.021*	.021*
α_{02}	$.13\times10^{-5}$	$.74\times10^{-6}$	$.53\times10^{-6}$
α_{12}	.949	.959	.961
α_{22}	.039	.028	.022
δ_{22}		.276	
δ_{42}		.173	
δ_{62}		.033	
δ_{82}		$.29\times10^{-2}$	
ρ_{12}	−.34	−.295	−.35
v			4.82
Log L	25720.1	25880.8	25855.5

* The parameter is not significant at 5% confidence level.
[a] ϕ_{0i} and ϕ_{1i} are the parameters of the AR(1); α_{0i}, α_{1i}, and α_{2i} are the GARCH(1,1) parameters; δ_{2i}, δ_{4i}, δ_{6i} and δ_{8i} are the parameters of the SNP; ρ_{12} is the correlation coefficient; n represents the degrees of freedom of the Student's t; LogL stands for the log-likelihood.

Once the conditional first and second moments were estimated consistently with the SNP hypothesis, we proceeded with the estimation of the parameters of the portfolio distribution (δ_{2y}, δ_{4y}, δ_{6y} and δ_{8y} parameters) constrained to the estimated portfolio mean and variance – equations (7) and (8).

Tables II, III and IV displays the estimates for the portfolio distribution and the corresponding VaRs at different confidence levels (α = 0.05, α = 0.025 and α = 0.01) and weighting Dow Jones index by $\theta_1 = 0.1$, $\theta_1 = 0.5$ and $\theta_1 = 0.9$, respectively. These results highlight the fact that the underestimation of VaR under the normality assumption is higher the lower the confidence level (e.g. for $\theta_1 = 0.1$ and α = 0.05 the VaR obtained under the SNP hypothesis are just 0.7% bigger than the Gaussian VaR but the difference increases to the 11.3% for the same portfolio and α = 0.01). Moreover, for such confidence levels VaR estimates are significantly bigger when assuming a Student's t than an SNP distribution. This evidence is consistent with [18] who found that the degrees of freedom of the Student's t could be understated when fitting high frequency densities since this parameter must not only capture the thickness at the tails but also the sharp peak at the mean. In consequence, the VaR obtained under the Student's t might be overestimated (particularly for high confidence levels). Note that the estimates for the degrees of freedom of Student's t (denoted by v in the tables) are less than 4 in some cases, which implies the non existence of the fourth order moment. On the other hand, these problems do not occur under the SNP distribution, where tails behavior is captured jointly by several parameters and moments of all order exist.

TABLE II

PORTFOLIO DISTRIBUTION ESTIMATES[a] AND VaR (θ_1=.1) [b]

	NOR	SNP	ST
δ_{2p}		.162	
δ_{4p}		.142	
δ_{6p}		.024	
δ_{8p}		$.2\times10^{-2}$	
v			4.46
σ_y	$.72\times10^{-2}$	$.6\times10^{-2}$	$.58\times10^{-2}$
VaR .05[c]	.0118	.0119	.0154
% .05		**0.7**	**30.4**
VaR .025	.0141	.0153	.0192
% .025		**8.8**	**36.6**
VaR .01	.0167	.021	0.024
% .01		**11.3**	**48.4**

[a] δ_{2p}, δ_{4p}, δ_{6p} and δ_{8p} are the parameters of portfolio under the SNP distribution; n represents the degrees of freedom of the Student's t for the portfolio; σ_y stands for the portfolio variance.
[b] Weight for the Dow Jones index.
[c] VaR(α) stands for the VaR at confidence level α; %(α) represents the difference of the VaR under either SNP or Student's t and the VaR implied by the normality hypothesis (in percentage).

TABLE III

PORTFOLIO DISTRIBUTION ESTIMATES[a] AND VaR (θ_1=.5) [b]

	NOR	SNP	ST
δ_{2p}		.089	
δ_{4p}		.088	
δ_{6p}		.014	
δ_{8p}		$.2\times10^{-2}$	
v			3.66
σ_y	$.44\times10^{-2}$	$.4\times10^{-2}$	$.35\times10^{-2}$
VaR .05[c]	.0073	.0079	.0102
% .05		**7.6**	**40.0**
VaR .025	.0087	.0097	.0131
% .025		**12.4**	**50.6**
VaR .01	.0103	.012	0.017
% .01		**20.0**	**71.0**

[a] δ_{2p}, δ_{4p}, δ_{6p} and δ_{8p} are the parameters of portfolio under the SNP distribution; n represents the degrees of freedom of the Student's t for the portfolio; σ_y stands for the portfolio variance.
[b] Weight for the Dow Jones index.
[c] VaR(α) stands for the VaR at confidence level α; %(α) represents the difference of the VaR under either SNP or Student's t and the VaR implied by the normality hypothesis (in percentage).

TABLE IV

PORTFOLIO DISTRIBUTION ESTIMATES[a] AND VaR (θ_1=.9) [b]

	NOR	SNP	ST
δ_{2p}		.099	
δ_{4p}		.013	
δ_{6p}		.212	
δ_{8p}		$.2\times10^{-2}$	
v			3.02
σ_y	$.63\times10^{-2}$	$.6\times10^{-2}$	$.49\times10^{-2}$
VaR .05[c]	.0104	.0118	.0156
% .05		**13.7**	**32.7**
VaR .025	.0123	.0152	.0205
% .025		**22.87**	**66.6**
VaR .01	.0146	.0205	0.028
% .01		**40.1**	**96.0**

[a] δ_{2p}, δ_{4p}, δ_{6p} and δ_{8p} are the parameters of portfolio under the SNP distribution; n represents the degrees of freedom of the Student's t for the portfolio; σ_y stands for the portfolio variance.
[b] Weight for the Dow Jones index.
[c] VaR(α) stands for the VaR at confidence level α; %(α) represents the difference of the VaR under either SNP or Student's t and the VaR implied by the normality hypothesis (in percentage).

Finally, it is also noteworthy that this approach allows the analysis of the sensitivity of the VaR to changes in relation to the portfolio weights. For example, in Table II it is easy to check that the portfolio VaR increases as the weight of the long run interest rates decreases. Therefore, the estimates of different portfolios can help risk managers to choose those strategies that minimize VaR at any period depending on weights and confidence levels.

IV. CONCLUSION

SNP distributions have been shown capable of fitting financial univariate densities more accurately than other popular densities used in finance such as the Student's t ([11] provide clear evidence of this issue). These flexible SNP distributions can also be generalized to a multivariate context and thus used to estimate the whole distribution of portfolio variables. This article proposes to calculate portfolio's VaR consistently with the SNP distributional hypothesis in a three-step procedure: In the first step the variance and covariance matrix is jointly estimated in the multivariate SNP density**.** In the second step, portfolio variance is computed and the (univariate) portfolio distribution is estimated constrained to the previous estimation of the portfolio variance; And in the third step the quantiles of the portfolio SNP distribution are worked out given the chosen confidence level. For the sake of comparison, the analyses are carried out under the Normal, the SNP and the Student's t distribution. Finally, we summarize the main conclusions of the study:

1.- VaR methodologies should incorporate the non-normality of most high frequency financial variables. The SNP distributions are capable of accounting for the main empirical features of most high frequency data and provide a flexible and simple parametric representation of the underlying (true) density. Moreover, these distributions generalize the Normal and, therefore, their comparison to the Normal can be straightforwardly done in terms of the traditional non-linear restriction tests or accuracy criteria.

2.- The variance and covariance matrix of portfolio variables can be estimated consistently with the SNP specifications, since SNP densities can be generalized to a multivariate context in a natural way (i.e. marginals remain within the same family). Additionally, different time varying variance hypotheses, such as a GARCH or stochastic volatility models (see [24] for a comprehensive survey on the latter models), can be also implemented.

3.- Portfolio VaRs computed under the SNP distribution seem to be higher than the VaR under normality, but lower than the VaR obtained by assuming the Student's t distribution. This means a clear underestimation of the VaR when only the first and second order moments are used (i.e. under normality) and, probably, an overestimation of the VaR when the Student's t is used. This assessment is explained by the underestimation of the degrees of freedom of this distribution (in an attempt to capture both the sharp peak and the thick tails with the same parameter).

4.- The flexible specification of the SNP distribution may be an interesting tool for risk management (see e.g. [23]) since VaR measures can be improved by considering not only the conditional variance, but also higher order moments. Within this framework, an interesting approach would be the search of those strategies that minimize VaR at any period, depending on weights and confidence levels.

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