Calculation of all stabilizing PI and PID controllers

Radek Matušů

***Abstract*—This paper deals with calculation of all stabilizing Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) controllers. The stability region, representing the area of possible placement of the controller parameters which guarantee feedback stabilization of a controlled plant, is obtained via plotting the stability boundary locus in the P-I plane or the P-I-D space by means of the Tan's method or the Kronecker summation method. These approaches are subsequently extended in order to compute robustly stabilizing PI controllers for interval plants. Moreover, the stabilization techniques are combined with the desired model method which is used for final controller design. The applicability of the methods is demonstrated on three control examples.

***Keywords*—Desired model method, Kronecker summation method, linear control, PI controllers, PID controllers, stability regions, stabilization, robust stabilization, Tan's method.

I. Introduction

Over 95% of contemporary practical industrial applications use PID (or PI as a special case) control algorithms [1]–[7] and thus the appropriate PI(D) control design is still very topical especially for systems under some nonlinearities, perturbations or time-variant behaviour. Without any doubts, the absolutely primary and essential requirement of all applications is the stability of closed control loop.

There is an array of techniques to computation of stabilizing PI(D) controllers in the literature such as calculations presented in [8], the Tan's method from [9], [10] or the Kronecker summation method published in [11]. Moreover, these techniques have been also extended for robust stabilization of interval plants through combination with the sixteen plant theorem [12], [13]. However, all those tools solve "only" the problem of finding the area of all possible stabilizing or robustly stabilizing variations of PI(D) controller parameters. For the control design itself, potentially with additional safety or performance specifications, another method has to be utilized. For the purpose of this paper, the desired model method, formerly known as the inversion dynamics method, has been applied [14].

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This research was supported by the Ministry of Education, Youth and Sports of the Czech Republic under the Research Plan No. MSM 7088352102 and by the European Regional Development Fund under the project CEBIA-Tech No. CZ.1.05/2.1.00/03.0089. This assistance is very gratefully acknowledged.

This paper is the extended version of the contribution [15] and presents the PI and PID controller design using the combination of preliminary stability regions computation via stability boundary locus approach (the Tan's method or the Kronecker summation method) and consequent suitable parameters tuning with the assistance of the desired model method. On the top of that, the paper outlines an approach to design of robustly stabilizing PI controllers based on the combination of the stability boundary locus computation and the sixteen plant theorem. The efficiency of the studied techniques has been verified through a trio of simulation examples where the various models have been successfully stabilized or robustly stabilized.

The work is organized as follows. In Section II, the basic ideas and rules for computation of stabilizing regions for PI controllers are described. The Section III than follows the previous one with the illustrative example. Further, the calculation of stability regions for PID controllers is presented in Section IV with supplementing example in Section V. Subsequently, Section VI provides the basics of robust stabilization using PI controllers and, again, Section VII contains accompanying example. And finally, Section VIII offers some conclusion remarks.

II. Computation of Stability Regions for PI Controllers

Assume the classical and very well known feedback control system shown in Fig. 1, where $C(s)$ is a controller, $G(s)$ represents a controlled system, and signals $w(t)$, $e(t)$, $u(t)$ and $y(t)$ denote a reference value, tracking (control) error, actuating (control) signal and output (controlled) variable, respectively.

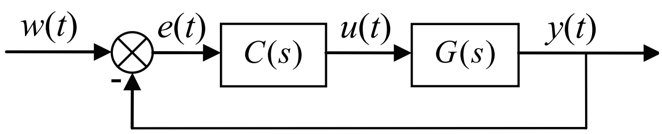

Fig. 1 closed-loop control system

The primary and essential step is to determine the parameters of a controller which guarantee stabilization of this feedback loop containing the plant:

$$G(s)=\frac{B(s)}{A(s)} \tag{1}$$

First, the case of PI controller given by transfer function:

$$C(s)=k_P+\frac{k_I}{s}=\frac{k_P s+k_I}{s} \tag{2}$$

is considered.

One of the possible approaches to computation of stabilizing PI controllers, the Tan's method, has been published in [9], [10]. It is based on plotting the stability boundary locus. The substitution $s=j\omega$ in the function (1) and subsequent decomposition of the numerator and denominator into their even and odd parts lead to:

$$G(j\omega)=\frac{B_E(-\omega^2)+j\omega B_O(-\omega^2)}{A_E(-\omega^2)+j\omega A_O(-\omega^2)} \tag{3}$$

Then, the expression of closed-loop characteristic polynomial and equaling the real and imaginary parts to zero result in the relations for proportional and integral gains:

$$\begin{aligned} k_P(\omega)&=\frac{P_5(\omega)P_4(\omega)-P_6(\omega)P_2(\omega)}{P_1(\omega)P_4(\omega)-P_2(\omega)P_3(\omega)}\\ k_I(\omega)&=\frac{P_6(\omega)P_1(\omega)-P_5(\omega)P_3(\omega)}{P_1(\omega)P_4(\omega)-P_2(\omega)P_3(\omega)} \end{aligned} \tag{4}$$

where

$$\begin{aligned} P_1(\omega)&=-\omega^2 B_O(-\omega^2)\\ P_2(\omega)&=B_E(-\omega^2)\\ P_3(\omega)&=\omega B_E(-\omega^2)\\ P_4(\omega)&=\omega B_O(-\omega^2)\\ P_5(\omega)&=\omega^2 A_O(-\omega^2)\\ P_6(\omega)&=-\omega A_E(-\omega^2) \end{aligned} \tag{5}$$

Simultaneous solution of equations (4) and plotting the obtained values into the (k_P,k_I) plane define the stability boundary locus. The obtained curve together with the line $k_I=0$ split the (k_P,k_I) plane into the stable and unstable regions. The decision if the respective region represents stabilizing or unstabilizing area can be done using a test point within each region. Nevertheless, the appropriate frequency gridding is a potential problem. Thus, the Nyquist plot based technique from [8] can be used for improvement of the method. In this embellishment, the frequency can be separated into several intervals within which the stability or instability can not change. The borders of such intervals are defined by the real values of ω which fulfill the equation:

$$\operatorname{Im}\left[G(s)\right]=0 \tag{6}$$

Certainly, obtained intervals are very helpful for the proper frequency scaling.

As it has been already indicated, the Tan's approach from [9], [10] is not the only possible. For example, an alternative way, known as the Kronecker summation method, has been published in [11]. It is based on interesting properties of the Kronecker sum operation.

The work [11] has proved that each couple (k_P,k_I) satisfying:

$$\det\left(M\oplus M\right)=0 \tag{7}$$

determines the boundary of stability. The symbol $\oplus$ stands for Kronecker summation [16] and M is a square matrix:

$$M=\begin{bmatrix} 0 & 1 & 0 & 0\ \cdots & 0\\ 0 & 0 & 1 & 0\ \cdots & 0\\ \vdots & 0 & 0 & 1\ \cdots & \vdots\\ \vdots & \vdots & \vdots & \vdots\ \ddots & 0\\ 0 & 0 & 0 & \cdots\ 0 & 1\\ -\frac{f_0(k_P,k_I)}{f_n(k_P,k_I)} & -\frac{f_1(k_P,k_I)}{f_n(k_P,k_I)} & -\frac{f_2(k_P,k_I)}{f_n(k_P,k_I)} & \cdots\ \cdots & -\frac{f_{n-1}(k_P,k_I)}{f_n(k_P,k_I)} \end{bmatrix} \tag{8}$$

where the coefficients in the last row follow from the characteristic polynomial of the closed-loop connection from Fig. 1:

$$\begin{aligned} P_{CL}(s)&=A(s)s+B(s)\left(k_P s+k_I\right)=\\ &=f_n(k_P,k_I)s^n+\cdots+f_1(k_P,k_I)s+f_0(k_P,k_I) \end{aligned} \tag{9}$$

Analogically to the Tan's method, the determination of the stabilizing areas can be done using a test point within each region.

The computation process is going to be illustrated in the following example.

III. Example I – Stabilizing PI Control

A. Calculation of all stabilizing PI controllers

Consider a third order controlled system from [17] given by transfer function:

$$G(s)=\frac{5}{s^3+2s^2+3s+4} \tag{10}$$

The aim is to compute all stabilizing PI controllers. First, the Tan's method has been applied. The even and odd parts in plant (3) are:

$$\begin{aligned} B_E(-\omega^2) &= 5 \\ B_O(-\omega^2) &= 0 \\ A_E(-\omega^2) &= 2(-\omega^2)+4 \\ A_O(-\omega^2) &= -\omega^2+3 \end{aligned} \tag{11}$$

The relations (4) take the form:

$$\begin{aligned} k_P(\omega) &= 0.4\omega^2 - 0.8 \\ k_I(\omega) &= -0.2\omega^4 + 0.6\omega^2 \end{aligned} \tag{12}$$

and the suitable frequency interval for plotting the stability boundary locus can be pre-calculated according to (6) with outcome:

$$\omega \in (0; 1.7321) \tag{13}$$

Simultaneous solving the equations (12) and plotting the results into the (k_P, k_I) plane leads to the stability region from Fig. 2.

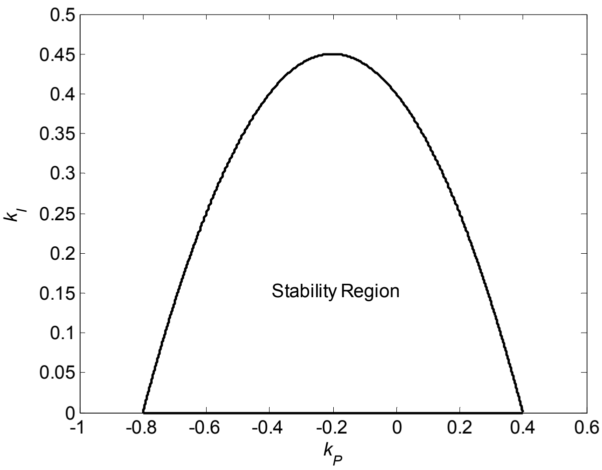

Fig. 2 stability region for system (10)

The fact that the inner space represents the area of stability can be simply verified using an arbitrary pair (k_P, k_I) from this region, calculating the corresponding closed-loop characteristic polynomial and testing its stability. The instability region then resides in the outer part which can be verified by the same procedure.

Alternatively, the very same result can be obtained via the Kronecker summation method. The closed-loop characteristic polynomial (9) takes the form:

$$P_{CL}(s) = s^4 + 2s^3 + 3s^2 + (5k_P + 4)s + 5k_I \tag{14}$$

Thus, the matrix constructed on the basis of (8) is:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5k_I & -5k_P-4 & -3 & -2 \end{bmatrix} \tag{15}$$

Next, one has to calculate the pairs (k_P, k_I) which fulfill the equality (7). They define the curve splitting the plane into the stable and unstable regions. Anyway, the final result is exactly the same as it has been already plotted in Fig. 2.

B. *Controller design*

Now, the natural question follows. How to choose the controller with desired performance from the pre-calculated stabilizing pool? In fact, the paper does not attempt to bring any novel control design method, but utilizes an existing one and combines it with the previous stabilizing approach. From the number of available techniques, the desired model method (formerly known as inversion dynamics method) [14] was applied.

First of all, the appropriate mathematical model of the controlled plant is requested in order to obtain PI controller. For that reason, the third order transfer function (10) can be simply approximated by the first order one as:

$$G_N(s) = \frac{1.25}{0.75s+1} = \frac{5}{3s+4} \approx \frac{5}{s^3+2s^2+3s+4} \tag{16}$$

which corresponds with the one of desired transfer function forms:

$$G_N(s) = \frac{K}{Ts+1}e^{-T_d s} \tag{17}$$

where:

$$\begin{aligned} K &= 1.25\,[-] \\ T &= 0.75\,[\mathrm{sec}] \\ T_d &= 0\,[\mathrm{sec}] \end{aligned} \tag{18}$$

The controller tuning is handled through the choice of closed control loop time constant T_w. Here, it was adjusted to:

$$T_w = 10\,[\mathrm{sec}] \tag{19}$$

The parameters of the controller:

$$C(s) = K_P\left(1 + \frac{1}{T_I s}\right) \tag{20}$$

can be calculated according to [14]:

$$T_I = T = 0.75 \qquad K_P = \frac{T_I}{KT_w} = 0.06 \tag{21}$$

Thus, the final parameters of the controller in the form (2) are:

$$k_P = K_P = 0.06 \qquad k_I = \frac{K_P}{T_I} = 0.08 \tag{22}$$

The Fig. 3 shows the position of PI controller with coefficients (22) in (k_P, k_I) plane. Thanks to the fact that this variation of parameters lies inside the stability region depicted in Fig. 2, the obtained PID controller stabilizes the plant (10).

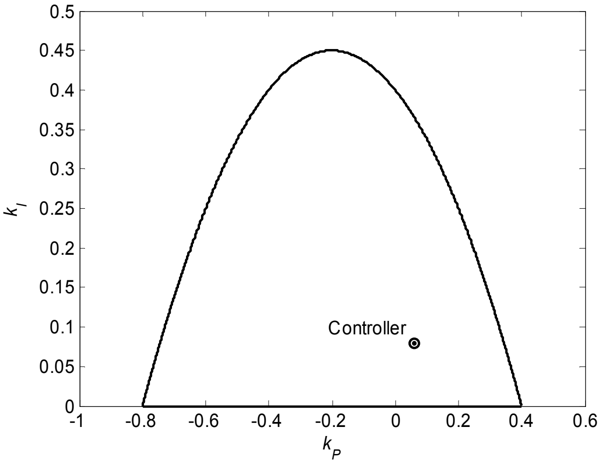

Fig. 3 position of the controller (22) in (k_P, k_I) plane

The actual control behaviour can be found in fig. 4.

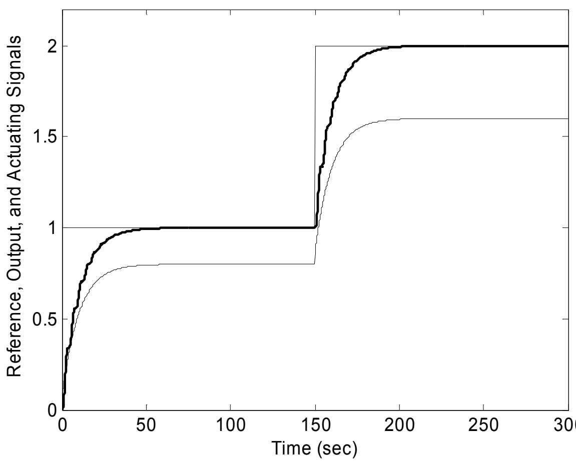

Fig. 4 control of plant (10) using PI controller (22)

IV. Computation of Stability Regions for PID Controllers

Now, the problem of closed-loop stabilization is going to be solved once more but for the PID controller case. All principal assumptions remain the same as in the Section II, only the feedback compensator in Fig. 1 now takes the form of ideal PID controller:

$$C(s) = k_P + \frac{k_I}{s} + k_D s = \frac{k_P s + k_I + k_D s^2}{s} \tag{23}$$

The analogical procedure presented in the Section II can be used for obtaining the stability boundary locus in the (k_P, k_I) plane for a fixed value of k_D. It leads to a bit modified equations for proportional and integral gains:

$$\begin{aligned} k_P(\omega, k_D) &= \frac{P_5(\omega)P_4(\omega) - P_6(\omega)P_2(\omega)}{P_1(\omega)P_4(\omega) - P_2(\omega)P_3(\omega)} \\ k_I(\omega, k_D) &= \frac{P_6(\omega)P_1(\omega) - P_5(\omega)P_3(\omega)}{P_1(\omega)P_4(\omega) - P_2(\omega)P_3(\omega)} \end{aligned} \tag{24}$$

where

$$\begin{aligned} P_1(\omega) &= -\omega^2 B_O(-\omega^2) \\ P_2(\omega) &= B_E(-\omega^2) \\ P_3(\omega) &= \omega B_E(-\omega^2) \\ P_4(\omega) &= \omega B_O(-\omega^2) \\ P_5(\omega) &= \omega^2 A_O(-\omega^2) + \omega^2 B_E(-\omega^2)k_D \\ P_6(\omega) &= -\omega A_E(-\omega^2) + \omega^3 B_O(-\omega^2)k_D \end{aligned} \tag{25}$$

Under new circumstances, the last two terms in (25) and thus also the parameters k_P and k_I (24) depend on derivative constant k_D, which is practically considered to be chosen and fixed for one set of calculations. In other words, k_D is preset and corresponding set of boundary parameters k_P, k_I is subsequently computed. The final stability region(s) are then successively plotted via the "(k_P, k_I) sections" in the (k_P, k_I, k_D) space.

Another possibility consists in computing the stability boundary locus in the (k_P, k_D) plane for a fixed value of k_I. This scenario changes the relations (24) and (25), respectively, to:

$$\begin{aligned} k_P(\omega, k_I) &= \frac{P_5(\omega)P_4(\omega) - P_6(\omega)P_2(\omega)}{P_1(\omega)P_4(\omega) - P_2(\omega)P_3(\omega)} \\ k_D(\omega, k_I) &= \frac{P_6(\omega)P_1(\omega) - P_5(\omega)P_3(\omega)}{P_1(\omega)P_4(\omega) - P_2(\omega)P_3(\omega)} \end{aligned} \tag{26}$$

and

$$\begin{aligned}
P_1(\omega) &= -\omega^2 B_O(-\omega^2)\\
P_2(\omega) &= -\omega^2 B_E(-\omega^2)\\
P_3(\omega) &= \omega B_E(-\omega^2)\\
P_4(\omega) &= -\omega^3 B_O(-\omega^2)\\
P_5(\omega) &= \omega^2 A_O(-\omega^2) - B_E(-\omega^2)k_I\\
P_6(\omega) &= -\omega A_E(-\omega^2) - \omega B_O(-\omega^2)k_I
\end{aligned} \tag{27}$$

Obviously, the final stability region(s) are given by the "(k_P, k_D) sections" in the (k_P, k_I, k_D) space.

However, the third option of obtaining the stability boundary – in the (k_I, k_D) plane for a fixed value of k_P – is not so straightforward, because then:

$$P_1(\omega)P_4(\omega) - P_2(\omega)P_3(\omega) = 0 \tag{28}$$

Nevertheless, the stability region in the (k_I, k_D) plane for a fixed k_P can be obtained using the stability region in the (k_P, k_I) plane and (k_P, k_D) plane together as it has been presented in [10]. In accordance with a linear programming based approach from [18], the stability region in the (k_I, k_D) plane under fixed k_P is a convex polygon.

Apart from the Tan's method, the stabilizing variations of k_P, k_I, k_D parameters can be calculated also with the assistance of the Kronecker summation method which has been already outlined in the Section II.

The specific example of calculation is given in the following part.

V. Example II – Stabilizing PID Control

A. Calculation of all stabilizing PI controllers

The electronic laboratory model has been considered as a controlled system. Its transfer function adopted from [19], [20] can be written as:

$$G(s) = \frac{2.925}{175.5s^3 + 137.5s^2 + 22s + 1} \tag{29}$$

It means that the even and odd parts from the transfer function (3) are:

$$\begin{aligned}
B_E(-\omega^2) &= 2.925\\
B_O(-\omega^2) &= 0\\
A_E(-\omega^2) &= 137.5(-\omega^2) + 1\\
A_O(-\omega^2) &= 175.5(-\omega^2) + 22
\end{aligned} \tag{30}$$

In the first instance, the derivative gain k_D was fixed to 1 and then the relations (25) and (24) were computed for a suitable range of nonnegative frequencies. The corresponding pairs of (k_P, k_I) are plotted in Fig. 5. The stabilizing area lies inside the depicted shape as can be easily verified using an arbitrary (k_P, k_I) from this region and testing the closed-loop characteristic polynomial stability.

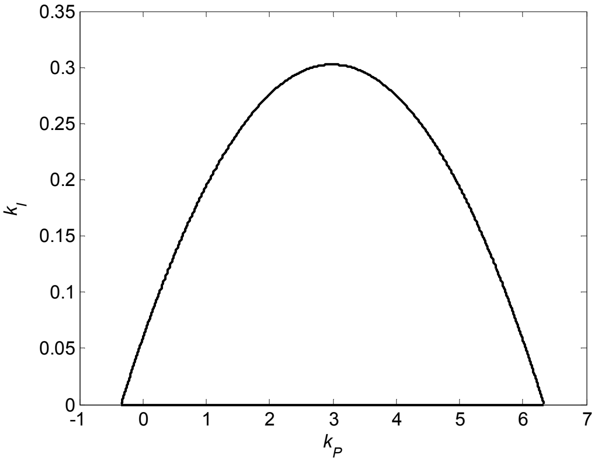

Fig. 5 stability region for system (29) and for $k_D = 1$

Afterward, the stability regions were computed and visualized for 11 equally spaced k_D from 0 to 10. The result is shown in Fig. 6.

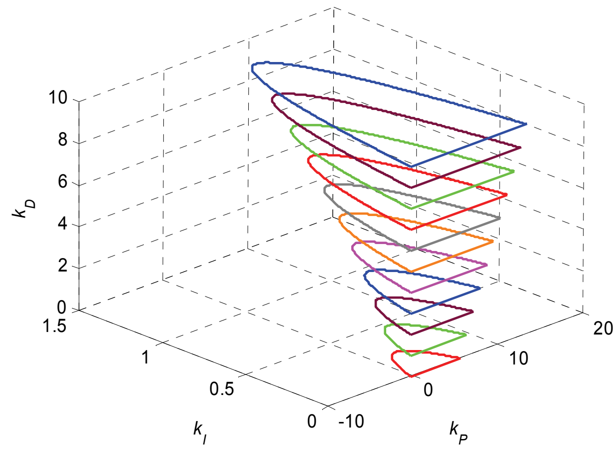

Fig. 6 stability regions for system (29) and for $k_D \in \langle 0, 10 \rangle$

Thus, all the variations of PID controller parameters which are located inside the shape defined by stability regions from Fig. 6 ensure the feedback stabilization of the plant (29).

An alternative approach based on cutting the sections in (k_P, k_D) plane for a fixed value of k_I is shown in Fig. 7, where 11 equally spaced k_I from 0 to 1 were assumed. As can be seen, the final 3-D stability regions from Figs. 6 and 7 cover the same areas.

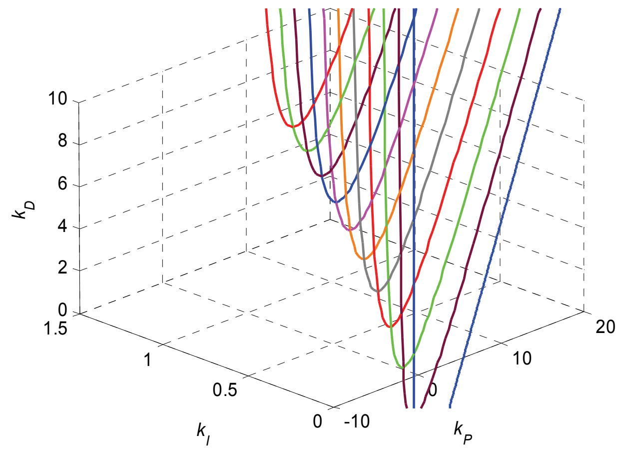

Fig. 7 stability regions for system (29) and for $k_I \in \langle 0,1 \rangle$

B. Controller design

The final selection of the controller from the stability region in Figs. 6 and 7 has been done by means of the desired model method [14], as in the previous example in Section III.

In order to design an ideal continuous-time PID controller, the requested mathematical model of the controlled plant can be obtained via simple approximation:

$$G_N(s) = \frac{2.925}{137.5s^2 + 22s + 1} \approx \frac{2.925}{175.5s^3 + 137.5s^2 + 22s + 1} \quad (31)$$

which concurs with desired transfer function:

$$G_N(s) = \frac{K}{T^2 s^2 + 2\xi T s + 1} e^{-T_d s}, \quad 0.5 < \xi \le 1 \quad (32)$$

where:

$$K = 2.925\,[-], \quad T = 11.726\,[\text{sec}], \quad \xi = 0.9381\,[-], \quad T_d = 0\,[\text{sec}] \quad (33)$$

Now, the controller is tuned through the selection of closed control loop time constant:

$$T_w = 20\,[\text{sec}] \quad (34)$$

The coefficients of the PID controller:

$$C(s) = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) \quad (35)$$

have been computed as [14]:

$$T_I = 2\xi T = 22, \quad K_P = \frac{T_I}{K T_w} = 0.3761, \quad T_D = \frac{T}{2\xi} = 6.25 \quad (36)$$

Consequently, the parameters of the controller applicable to the transfer function (23) are:

$$k_P = K_P = 0.3761, \quad k_I = \frac{K_P}{T_I} = 0.0171, \quad k_D = K_P T_D = 2.3504 \quad (37)$$

As such variation of parameters is located inside the stability region from Figs. 6 and 7, the corresponding PID controller must stabilize the system (29). The Fig. 3 shows the control results.

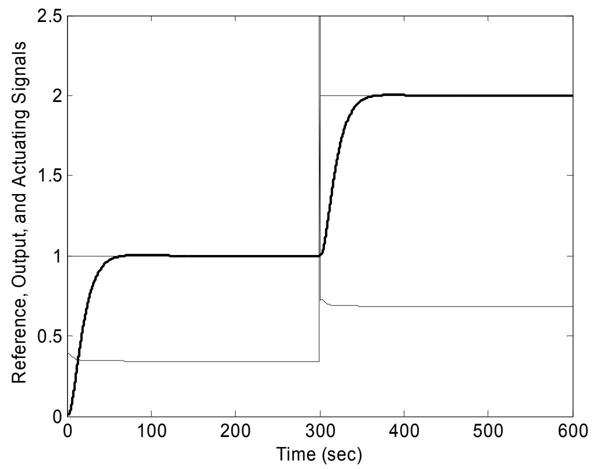

Fig. 8 control of plant (6) using PID controller (14)

VI. Robust Stabilization Using PI Controller

So far, the stabilization of systems with fixed parameters has been the object of interest. Nevertheless, the works [9], [10], [11] have embellished an arbitrary feedback stabilization technique also for systems whose coefficients can vary within given intervals, i.e. for interval plants, simply by using its combination with the sixteen plant theorem [12], [13], [21]. Thus, this improvement leads to a tool for possible robust stabilization of the whole family given by interval plant using the single fixed PI controller. The sixteen plant theorem says that a first order controller robustly stabilizes an interval plant:

$$G(s,b,a) = \frac{B(s,b)}{A(s,a)} = \frac{\sum_{i=0}^{m}\left[b_i^-, b_i^+\right]s^i}{s^n + \sum_{i=0}^{n-1}\left[a_i^-, a_i^+\right]s^i}; \quad m < n \quad (38)$$

where $b_i^-, b_i^+, a_i^-, a_i^+$ represent lower and upper bounds for parameters of numerator and denominator, if and only if it stabilizes its 16 Kharitonov plants, defined as:

$$G_{i,j}(s) = \frac{B_i(s)}{A_j(s)} \tag{39}$$

where $i, j \in \{1,2,3,4\}$; and $B_1(s)$ to $B_4(s)$ and $A_1(s)$ to $A_4(s)$ are the Kharitonov polynomials for the numerator and denominator of the interval plant (38).

Remind that the construction of Kharitonov polynomials e.g. for the numerator interval polynomial:

$$B(s,b) = \sum_{i=0}^{m} \left[b_i^-; b_i^+ \right] s^i \tag{40}$$

is based on use of the lower and upper bounds of interval parameters in compliance with the principle [22]:

$$\begin{aligned} B_1(s) &= b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + \cdots \\ B_2(s) &= b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + \cdots \\ B_3(s) &= b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + \cdots \\ B_4(s) &= b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + \cdots \end{aligned} \tag{41}$$

Consequently, the robust stabilization of an interval plant directly follows from the simultaneous stabilization of all 16 fixed Kharitonov plants. Thus, the final area of stability for original interval plant is given by the intersection of all 16 related partial areas obtained individually using the techniques from the Section II. All this process is going to be elucidated in the following example.

VII. Example III – Robustly Stabilizing PI Control

A. *Calculation of all robustly stabilizing PI controllers*

Suppose the controlled process described by third order transfer function with interval parameters, which is adopted from [12], [20]:

$$G(s,b,a) = \frac{[0.75, 1.25]s + [0.75, 1.25]}{s^3 + [2.75, 3.25]s^2 + [8.75, 9.25]s + [0.75, 9.25]} \tag{42}$$

Assembling the 16 Kharitonov plants according to (39), e.g.:

$$G_{1,1}(s) = \frac{0.75s + 0.75}{s^3 + 3.25s^2 + 8.75s + 0.75} \tag{43}$$

and computing/plotting all 16 corresponding stability regions lead to the final stability region determined by their intersection. The result is visualized in Fig. 9, where the open highlighted area represents the suitable variations of PI controller parameters which robustly stabilize the interval plant (42).

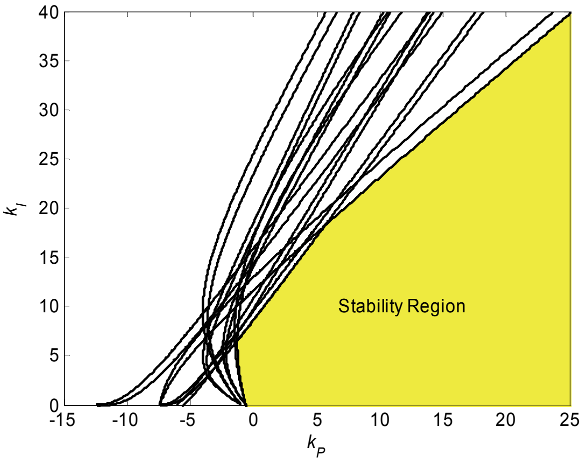

Fig. 9 stability region for the interval plant (42)

B. *Controller design*

Again, the desired model method [14] has been utilized for final controller tuning. Computing the average values of interval parameters in transfer function (42) and subsequent approximation by the first order system results in the nominal system, which similarly to the example in Section III fits one of the desired models:

$$G_N(s) = \frac{1}{9s+5} = \frac{0.2}{1.8s+1} = \frac{K}{Ts+1} \tag{44}$$

The closed control loop time constant has been selected:

$$T_w = 1\,[\mathrm{sec}] \tag{45}$$

which influences the parameters of the controller (20):

$$\begin{aligned} T_I &= T = 1.8 \\ K_P &= \frac{T_I}{KT_w} = 9 \end{aligned} \tag{46}$$

Actually:

$$\begin{aligned} k_P &= K_P = 9 \\ k_I &= \frac{K_P}{T_I} = 5 \end{aligned} \tag{47}$$

Such parameters reside in the stability region from Fig. 9 and thus the closed loop system with the PI controller (47) and interval plant (42) is robustly stable. The control results from Fig. 10 visually confirm the robust stability. It shows the control responses of the loop with the controller (47) and 1024 "representative" systems from the interval family (42). Each interval parameter has been divided into 3 subintervals and thus these 4 values and 5 parameters result in $4^5 = 1024$ systems for simulation. Moreover, the red curve represents the output variable for the nominal system (44).

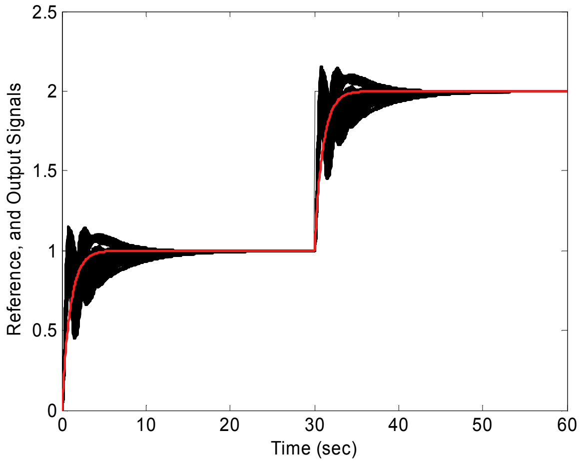

Fig. 10 output signals of 1024 "representative" plants and 1 nominal system

Such control result should be appropriate for most of common industrial applications.

VIII. Conclusion

The paper has dealt with computation of stability regions for PI and PID controllers. The combination of pre-computing the stabilizing areas and designing the PI(D) controller which lies inside it represents relatively easy but effective way of obtaining the stabilizing PID controller with acceptable performance. Furthermore, the stabilization techniques were extended by means of robust stabilization of interval plants using PI controllers.

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