Analysis of feeding device with two degrees of freedom

Jiri Balla, Van Yen Duong

***Abstract*—**The paper deals with kinematic and dynamic analysis of the feeding device mounted on the Czech 152 mm Self-propelled Howitzer mod. 77 (152 mm SPH M77). Kinematic analysis of the feeding device has been worked out using MATLAB software. The dynamic problem has been solved using Lagrange multiplier method applied for closed chain multi-body system. After that both kinematic and dynamic tasks has been simulated using of Rigid Dynamic component of software ANSYS with Finite Element Method (FEM).

***Keywords*—**Closed chain robot, Feeding mechanism, Gun loading, Kinematic analysis, Dynamic simulation, Matrix method, Working space, Feeding device, Multi-body system, Lagrange multiplier function.

I. Introduction

THE feeding device with two degrees of freedom is used in the Czech 152 mm self-propelled cannon howitzer Mod. 77 - see Fig. 1 (view in the horizontal plane) and Fig. 2 (view in the vertical plane), see [2], [4], and [5]. The feeding device for projectiles and the feeding device for propellant charges in cases are mounted to the right and left cradle shafts, see Fig. 3. Their construction enables the feeding at any value of elevation, see [4], [9], [10], and [20]. After releasing of the feeder loading tray the feeder arm rotates and grips the projectile in the conveyer – the angle $\Delta\varphi_{52}^{1}$. Then it rotates in the opposite direction which describes the angle $\Delta\varphi_{52}^{2}$ into the vertical plane going through the barrel axis. Afterwards the loading tray is rotated by means of the rotating arm into the position when the direction of the barrel axis equals to the direction of the axis of the shell or the case – the angle φ_{21}.

The rotation in the horizontal plane (in the range $\Delta\varphi_{52}$) is possible using of the linear hydraulic motor having the rack bar at the end of the piston with transformation to the rotation by the gear wheel. The hoisting mechanism for the second motion in the vertical plane is driven by the hydraulic linear motor and then it is necessary to know relations between kinematic parameters of the hydraulic motor piston rod and the rotation of the arm φ_{21}.

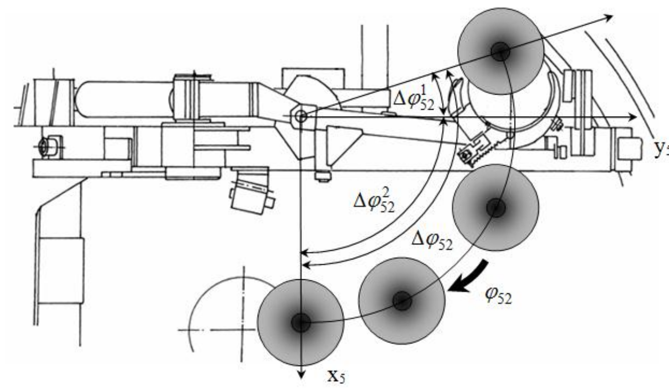

Fig. 1 View of the feeding device in the horizontal plane

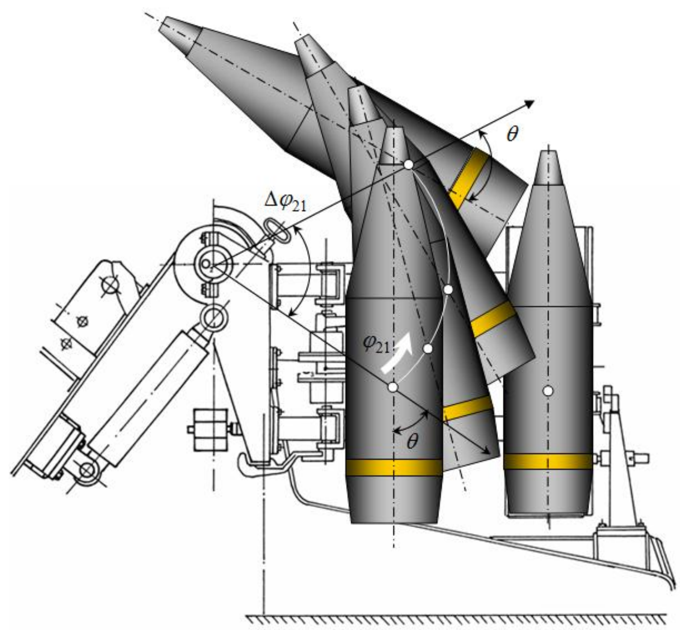

Fig. 2 View of the feeding device in the vertical plane

The feeding mechanism has the space positioning and the kinematic chain is formed with two rotation locomotor units and structure scheme is $R_x - R_z$ according to [7], [15], [16], and [18]. It means that the feeding device has revolute joints. Therefore only simple relationships describe the link transformations, see [18], [23], and [25]. In order to be solved this space problem; it is possible to perform the following steps.

Firstly, it is necessary to solve the both individual parts of

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J. Balla is with the University of Defense, Kounicova 65, 662 10 Brno, Czech Republic (corresponding author to provide phone: 00 420 973 44 5013; fax: 00 420 973 44 5318; e-mail: jiri.balla@ unob.cz).

V. Y. Duong is with the University of Defense, Kounicova 65, 662 10 Brno, Czech Republic (corresponding author to provide phone: 00 420 973 44 5163; fax: 00 420 973 44 5318; e-mail: yen.duongvan@gmail.com).

the feeding mechanism that drive moving sections in the vertical and horizontal plane. The vector method has been used for it in [2] and theory has been published in [7], and [18] for example.

Secondly, the outputs of the previous results are used as the inputs for the space problem of the feeding mechanism where the mechanics of manipulation equipment has been used.

Feeding process of the propellant charge is performed similarly to the projectile feeding, but by the left feeding device on the opposite side as it is shown in Fig. 3 where both devices are depicted.

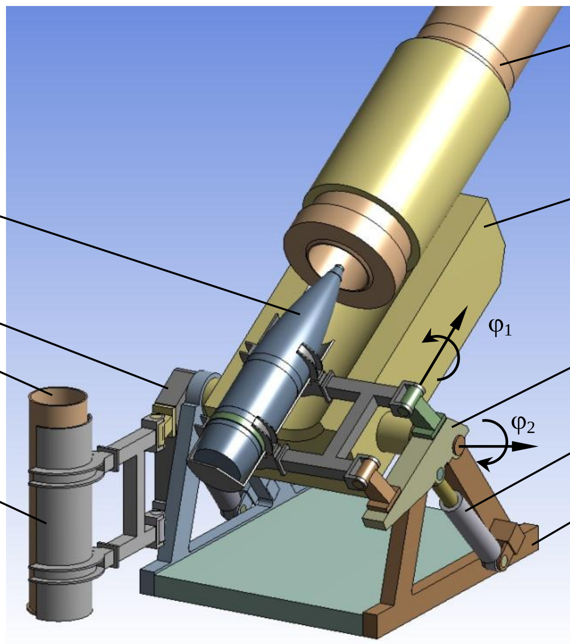

Fig. 3 Scheme of both feeding devices

The objects in Fig. 3 are: barrel - 1, elevation block - 2, projectile - 3, feeding device arms for propelling charge - 4 and for projectile - 7, driven component of feeding device (hydraulic piston- cylinder) - 5, frame in the upper carriage - 6, propellant charge in case – 8, feeding tray for case - 9.

According to [6], [15], [18], [21], and [22] have been defined the 4 x 4 transformation matrices for both rotation movements with respect to the translation of the origin of every coordinate system. These matrices are called homogeneous transformations and their advantageous is that they describe both position and orientation.

Fig. 4 describes the motion of the projectile during whole feeding process in course of the loading into the minimal elevation angle -4° when the trajectory is the largest, i.e. 94°. The trajectory of the propellant charge placed in own tray can be visualized by means of similar graph and the same way.

After the kinematic solution of the feeding device the dynamics follows to obtain reaction forces in connections between the kinematic elements.

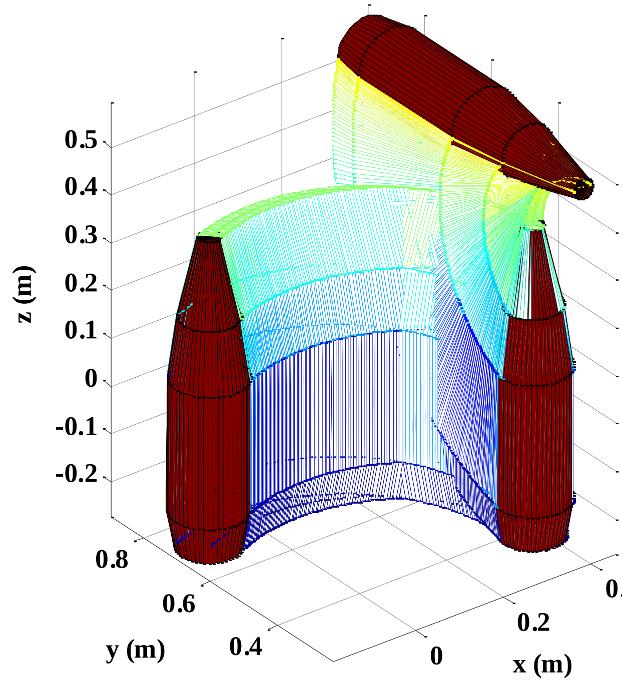

Fig. 4 Motion of the projectile during feeding

In the following section there will be given a summary of the mathematical background for establishing matrix form of dynamic function system of a closed chain robot using Lagrange multiplier function how it has been published in [6], [8], [17], and [26] with helping new attitude to the expressions of Lagrange equations with multipliers for constrained multibody systems.

II. Mathematical Background of Dynamics

Let us consider without demonstration the following lemmas:

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T \text{ - vector,}$$

$$\mathbf{x} \in \mathbb{R}^n,\ \alpha(\mathbf{x}) \text{ - function of variable x.}$$

Partial derivation of scalar function $\alpha(\mathbf{x})$ by vector variable $\mathbf{x}$ is defined as follows:

$$\frac{\partial \alpha}{\partial \mathbf{x}} = \partial \alpha \frac{1}{\partial \mathbf{x}^T} = \left[\frac{\partial \alpha}{\partial x_1} \quad \frac{\partial \alpha}{\partial x_2} \quad \dots \quad \frac{\partial \alpha}{\partial x_n} \right], \tag{1}$$

and

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \dots & a_m \end{bmatrix}^T \text{ - vector, } \mathbf{a}(\mathbf{x}) \in \mathbb{R}^m .$$

Then the partial derivative of function vector $\mathbf{a}$ with respect to the vector $\mathbf{x}$ is here defined as the following matrix:

$$\frac{\partial \mathbf{a}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial a_1}{\partial \mathbf{x}} \\ \frac{\partial a_2}{\partial \mathbf{x}} \\ \ldots \\ \frac{\partial a_m}{\partial \mathbf{x}} \end{bmatrix} = \begin{bmatrix} \frac{\partial a_1}{\partial x_1} & \frac{\partial a_1}{\partial x_2} & \ldots & \frac{\partial a_1}{\partial x_n} \\ \frac{\partial a_2}{\partial x_1} & \frac{\partial a_2}{\partial x_2} & \ldots & \frac{\partial a_2}{\partial x_n} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial a_m}{\partial x_1} & \frac{\partial a_m}{\partial x_2} & \ldots & \frac{\partial a_m}{\partial x_n} \end{bmatrix}. \qquad (2)$$

$\mathbf{A}(\mathbf{x}) \in \mathbb{R}^{m\times p}$ is function matrix, where $\mathbf{x} \in \mathbb{R}^n$ is vector. Matrix **A** can be presented as follows:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1p} \\ a_{21} & a_{22} & \ldots & a_{2p} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mp} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \ldots & \mathbf{a}_p \end{bmatrix}. \qquad (3)$$

Partial derivation of function matrix **A** by vector variable **x** is defined as follows, see [17]:

$$\frac{\partial \mathbf{A}(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial \mathbf{a}_1}{\partial \mathbf{x}} & \frac{\partial \mathbf{a}_2}{\partial \mathbf{x}} & \ldots & \frac{\partial \mathbf{a}_p}{\partial \mathbf{x}} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{a}_{11}}{\partial \mathbf{x}} & \frac{\partial \mathbf{a}_{12}}{\partial \mathbf{x}} & \ldots & \frac{\partial \mathbf{a}_{1\mathrm{p}}}{\partial \mathbf{x}} \\ \frac{\partial \mathbf{a}_{21}}{\partial \mathbf{x}} & \frac{\partial \mathbf{a}_{22}}{\partial \mathbf{x}} & \ldots & \frac{\partial \mathbf{a}_{2p}}{\partial \mathbf{x}} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial \mathbf{a}_{m1}}{\partial \mathbf{x}} & \frac{\partial \mathbf{a}_{m2}}{\partial \mathbf{x}} & \ldots & \frac{\partial \mathbf{a}_{mp}}{\partial \mathbf{x}} \end{bmatrix} \qquad (4)$$

Let $\mathbf{A} \in \mathbb{R}^{m\times p}$, and $\mathbf{B} \in \mathbb{R}^{q\times s}$ are matrices and product of two matrices **C** is defined as follows:

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} \in \mathbb{R}^{m\times s};\ c_{ij} = \sum\nolimits_{k=1}^{p} a_{ik} b_{kj}\ . \qquad (5)$$

Let $\mathbf{B} \in \mathbb{R}^{q\times s}$ is matrix, a_{ij} is constant, i=1,…, m; j=1,…,p, then the matrix $\begin{bmatrix} a_{ij} & \mathbf{B} \end{bmatrix}$ is defined as follows:

$$\begin{bmatrix} a_{ij} & \mathbf{B} \end{bmatrix} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & \ldots & a_{1p}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & \ldots & a_{2p}\mathbf{B} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1}\mathbf{B} & a_{m2}\mathbf{B} & \ldots & a_{mp}\mathbf{B} \end{bmatrix} \in \mathbb{R}^{mq\times ps}. \qquad (6)$$

Let us denote $\mathbf{I}_n$ as the *n* x *n* identity matrix.

If $\mathbf{A} \in \mathbb{R}^{m\times p}$ and $\mathbf{B} \in \mathbb{R}^{q\times s}$ are matrices. Then Kronecker product of two matrices **A** and **B** is marked as $\mathbf{A} \otimes \mathbf{B}$. It is matrix that has size $mq \times ps$ and it is defined and written by the following formula, see [6], [8], [17], and [26]:

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{ij} & \mathbf{B} \end{bmatrix} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & \ldots & a_{1p}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & \ldots & a_{2p}\mathbf{B} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1}\mathbf{B} & a_{m2}\mathbf{B} & \ldots & a_{mp}\mathbf{B} \end{bmatrix} \in \mathbb{R}^{mq\times ps}. \qquad (7)$$

When $\mathbf{A} \in \mathbb{R}^{m\times p}$ and $\mathbf{B} \in \mathbb{R}^{q\times s}$ are function matrices of vector **x**. Then the derivation of product of two matrixes is defined:

$$\frac{\partial}{\partial \mathbf{x}}\left[\mathbf{A}(\mathbf{x}).\mathbf{B}(\mathbf{x})\right] = \frac{\partial \mathbf{A}(\mathbf{x})}{\partial \mathbf{x}}\left[\mathbf{B}(\mathbf{x}) \otimes \mathbf{I}_n\right] + \mathbf{A}(\mathbf{x})\frac{\partial \mathbf{B}(\mathbf{x})}{\partial \mathbf{x}}. \qquad (8)$$

The consequences following from from (8) are following:

If $\mathbf{A} \in \mathbb{R}^{m\times p}$, $\mathbf{b} \in \mathbb{R}^{p}$ and $\mathbf{x} \in \mathbb{R}^{n}$ then:

$$\frac{\partial}{\partial \mathbf{x}}\left[\mathbf{A}(\mathbf{x}).\mathbf{b}\right] = \frac{\partial \mathbf{A}(\mathbf{x})}{\partial \mathbf{x}}\left[\mathbf{b} \otimes \mathbf{I}_n\right] + \mathbf{A}(\mathbf{x})\frac{\partial \mathbf{b}}{\partial \mathbf{x}}. \qquad (9)$$

If $\mathbf{b} \in \mathbb{R}^{p}$, $\mathbf{x} \in \mathbb{R}^{n}$ and $\alpha(\mathbf{x})$ is scalar then:

$$\frac{\partial}{\partial \mathbf{x}}\left[\mathbf{b}(\mathbf{x}).\alpha(\mathbf{x})\right] = \frac{\partial \mathbf{b}(\mathbf{x})}{\partial \mathbf{x}}\left[\alpha(\mathbf{x}) \otimes \mathbf{I}_n\right] + \mathbf{b}(\mathbf{x})\frac{\partial \alpha(\mathbf{x})}{\partial \mathbf{x}}. \qquad (10)$$

If $\alpha(\mathbf{x})$ and $\beta(\mathbf{x})$ are scalars, then:

$$\frac{\partial}{\partial \mathbf{x}}\left[\alpha(\mathbf{x}).\beta(\mathbf{x})\right] = \frac{\partial \alpha(\mathbf{x})}{\partial \mathbf{x}}\left[\beta(\mathbf{x}) \otimes \mathbf{I}_n\right] + \alpha(\mathbf{x})\frac{\partial \beta(\mathbf{x})}{\partial \mathbf{x}}. \qquad (10)$$

If $\mathbf{a} \in \mathbb{R}^{m}$, $\mathbf{b} \in \mathbb{R}^{m}$ and $\mathbf{x} \in \mathbb{R}^{n}$, then:

$$\frac{\partial(\mathbf{a}^T\mathbf{b})}{\partial \mathbf{x}} = \frac{\partial \mathbf{a}^T}{\partial \mathbf{x}}\left[\mathbf{b} \otimes \mathbf{I}_n\right] + \mathbf{a}^T\frac{\partial \mathbf{b}}{\partial \mathbf{x}}. \qquad (11)$$

Using Kronecker product, partial derivation of function matrix by vector variable is defined as follows:

$$\frac{\partial \dot{\mathbf{A}}(\mathbf{x})}{\partial \mathbf{x}} = \frac{\partial \mathbf{A}(\mathbf{x})}{\partial \mathbf{x}}(\mathbf{I}_n \otimes \dot{\mathbf{x}}). \qquad (12)$$

III. Matrix Form of Lagrange Multiplier Function

Now mathematical background presented above will be used for establishing of the matrix form of the Lagrange multiplier function for a multi-body system.

The Lagrange multiplier function, see [17], [24] for example, can be written by using of the *T* kinetic energy and Π potential energy as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}}\right)^T - \left(\frac{\partial T}{\partial \mathbf{q}}\right)^T = \left(\frac{\partial \Pi}{\partial \mathbf{q}}\right)^T - \mathbf{J}_{\mathbf{f}}^T + \boldsymbol{\tau}. \qquad (13)$$

Let us consider multi-body system contains p rigid bodies, f degrees of freedom, r holonomic constraints, and s linear nonholonomic constraints. Position of the system is determined by m generalized coordinates $\mathbf{q} = (q_1, q_2, ..., q_m)$, where $f < m$. From holonomic constraints we receive r constraint equations in the following form:

$$f_i(q_1, q_2, ..., q_m) = 0 \;,\; i = 1, ..., r \,. \tag{14}$$

Linear nonholonomic constraints are presented by s following equations:

$$\sum a_{jk}\dot{q}_k + a_{j0} = 0 \,,\; j = 1, ..., s \,. \tag{15}$$

Number of independent generalized coordinates of the system is:

$n = m - r.$

Number of degrees of freedom of the system is:

$f = n - s = m - (r + s)$.

The kinetic energy of the system is as follows, see [7], and [16]:

$$T = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) \,, \tag{16}$$

where: $\mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{M}(\mathbf{q})\dot{\mathbf{q}}$.

The $\mathbf{M}(\mathbf{q})$ inertia matrix is square, symmetric. It is calculated as follows using the $\mathbf{J}_{Ti}$, $\mathbf{J}_{Ri}$ translational and Jacobian matrices, see [17], [25]:

$$\mathbf{M}(\mathbf{q}) = \sum_{i=1}^{p}(m_i\mathbf{J}_{Ti}^T\mathbf{J}_{Ti} + \mathbf{J}_{Ri}^T I_i \mathbf{J}_{Ri}) \,. \tag{17}$$

The Lagrange equation of nonholonomic constraint system is written as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}}\right)^T - \left(\frac{\partial T}{\partial \mathbf{q}}\right)^T = \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 - \mathbf{g}(\mathbf{q}) \tag{18}$$

where:

$$\mathbf{g}(\mathbf{q}) = \left(\frac{\partial \Pi}{\partial \mathbf{q}}\right)^T,$$

$$\mathbf{f}_1 = \begin{bmatrix}\tau_1 & ... & \tau_m\end{bmatrix}^T,\; \mathbf{f}_2 = -\lambda^T\frac{\partial \mathbf{f}}{\partial \mathbf{q}} = -\mathbf{J}_{\mathbf{f}}^T\lambda \,,\; \mathbf{f}_3 = -\mathbf{A}^T\boldsymbol{\mu} \,,$$

where: $\mathbf{A} = \begin{bmatrix}a_{ik}\end{bmatrix}$,

$\mathbf{f}_1$ is exterior force vector equivalent to generalized coordinates q_1, q_2, q_3, q_4.

The partial derivation of the kinetic energy in (18) with respect to the vector $\dot{\mathbf{q}}$ is given:

$$\frac{\partial T}{\partial \dot{\mathbf{q}}} = \frac{1}{2}\frac{\partial}{\partial \dot{\mathbf{q}}}(\dot{\mathbf{q}}^T\mathbf{b}) = \frac{1}{2}\left[\frac{\partial \dot{\mathbf{q}}^T}{\partial \dot{\mathbf{q}}}(\mathbf{b}\otimes\mathbf{I}_m) + \dot{\mathbf{q}}^T\frac{\partial \mathbf{b}}{\partial \dot{\mathbf{q}}}\right], \tag{19}$$

where

$$\frac{\partial \dot{\mathbf{q}}^T}{\partial \dot{\mathbf{q}}}(\mathbf{b}\otimes\mathbf{I}_m) = \mathbf{b}^T = (\mathbf{M}\dot{\mathbf{q}})^T = \dot{\mathbf{q}}^T\mathbf{M} \,,$$

$$\dot{\mathbf{q}}^T\frac{\partial \mathbf{b}}{\partial \dot{\mathbf{q}}} = \dot{\mathbf{q}}^T\frac{\partial}{\partial \dot{\mathbf{q}}}(\mathbf{M}(\mathbf{q})\dot{\mathbf{q}}) = \dot{\mathbf{q}}^T\left[\frac{\partial \mathbf{M}(\mathbf{q})}{\partial \dot{\mathbf{q}}}(\dot{\mathbf{q}}\otimes\mathbf{I}_m) + \mathbf{M}(\mathbf{q})\frac{\partial \dot{\mathbf{q}}}{\partial \dot{\mathbf{q}}}\right],$$

where: $\dfrac{\partial \mathbf{M}(\mathbf{q})}{\partial \dot{\mathbf{q}}} = 0$; $\dfrac{\partial \dot{\mathbf{q}}}{\partial \dot{\mathbf{q}}} = \mathbf{I}_m$.

Then the last member in (19) is:

$$\dot{\mathbf{q}}^T\frac{\partial \mathbf{b}}{\partial \dot{\mathbf{q}}} = \dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\mathbf{I}_m = \dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q}) \tag{20}$$

By substituting (20) into (19), we obtain:

$$\frac{\partial T}{\partial \dot{\mathbf{q}}} = \frac{1}{2}\frac{\partial}{\partial \dot{\mathbf{q}}}(\dot{\mathbf{q}}^T\mathbf{b}) = \frac{1}{2}\left[\dot{\mathbf{q}}^T\mathbf{M} + \dot{\mathbf{q}}^T\mathbf{M}\right] = \dot{\mathbf{q}}^T\mathbf{M} \,, \tag{21}$$

$$\left(\frac{\partial T}{\partial \dot{\mathbf{q}}}\right)^T = \mathbf{M}(\mathbf{q})\dot{\mathbf{q}} \;;\; \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}}\right)^T = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{M}}(\mathbf{q})\dot{\mathbf{q}} \,. \tag{22}$$

The second component used in the derivation of the kinetic energy with respect to $\mathbf{q}$ is determined by next way:

$$\frac{\partial T}{\partial \mathbf{q}} = \frac{1}{2}\frac{\partial}{\partial \mathbf{q}}(\dot{\mathbf{q}}^T\mathbf{b}) = \frac{1}{2}\left[\frac{\partial \dot{\mathbf{q}}^T}{\partial \mathbf{q}}(\mathbf{b}\otimes\mathbf{I}_m) + \dot{\mathbf{q}}^T\frac{\partial \mathbf{b}}{\partial \mathbf{q}}\right]$$

$$= \frac{1}{2}\dot{\mathbf{q}}^T\frac{\partial}{\partial \mathbf{q}}(\mathbf{M}(\mathbf{q})\dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T\left[\frac{\partial \mathbf{M}(\mathbf{q})}{\partial \mathbf{q}}(\dot{\mathbf{q}}\otimes\mathbf{I}_m) + \mathbf{M}(\mathbf{q})\frac{\partial \dot{\mathbf{q}}}{\partial \mathbf{q}}\right]$$

$$= \frac{1}{2}\dot{\mathbf{q}}^T\frac{\partial \mathbf{M}(\mathbf{q})}{\partial \mathbf{q}}(\dot{\mathbf{q}}\otimes\mathbf{I}_m) \,. \tag{23}$$

$$\left(\frac{\partial T}{\partial \mathbf{q}}\right)^T = \left(\frac{1}{2}\dot{\mathbf{q}}^T\frac{\partial \mathbf{M}(\mathbf{q})}{\partial \mathbf{q}}(\dot{\mathbf{q}}\otimes\mathbf{I}_m)\right)^T = \frac{1}{2}\left[\frac{\partial \mathbf{M}(\mathbf{q})}{\partial \mathbf{q}}(\dot{\mathbf{q}}\otimes\mathbf{I}_m)\right]^T\dot{\mathbf{q}} \,. \tag{24}$$

By substituting (22), (24) into (18) we receive the following formula:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{M}}(\mathbf{q})\dot{\mathbf{q}} - \frac{1}{2}\left[\frac{\partial \mathbf{M}(\mathbf{q})}{\partial \mathbf{q}}(\dot{\mathbf{q}}\otimes\mathbf{I}_m)\right]^T\dot{\mathbf{q}} + \left(\frac{\partial \Pi}{\partial \mathbf{q}}\right)^T =$$

$$= \boldsymbol{\tau} - \mathbf{J}_{\mathrm{f}}^T - \mathbf{A}^T\boldsymbol{\mu} \,. \tag{25}$$

$$\mathbf{M(q)}\ddot{\mathbf{q}}+\left\{\dot{\mathbf{M}}(\mathbf{q})-\frac{1}{2}\left[\frac{\partial\mathbf{M(q)}}{\partial\mathbf{q}}(\dot{\mathbf{q}}\otimes\mathbf{I}_m)\right]^T\right\}\dot{\mathbf{q}}+\left(\frac{\partial\Pi}{\partial\mathbf{q}}\right)^T=$$

$$=\boldsymbol{\tau}-\mathbf{J}_{\mathrm{f}}^T-\mathbf{A}^T\boldsymbol{\mu}\,. \tag{26}$$

Let us note, see [17]:

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})=\frac{\partial\mathbf{M(q)}}{\partial\mathbf{q}}(\mathbf{I}_m\otimes\dot{\mathbf{q}})-\frac{1}{2}\left[\frac{\partial\mathbf{M(q)}}{\partial\mathbf{q}}(\dot{\mathbf{q}}\otimes\mathbf{I}_m)\right]^T. \tag{27}$$

Finally by substituting (27) into (26) we receive the motion equation in matrix form:

$$\mathbf{M(q)}\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g(q)}=\boldsymbol{\tau}-\mathbf{J}_{\mathrm{f}}^T-\mathbf{A}^T\boldsymbol{\mu}\,. \tag{28}$$

IV. Establishing Dynamic Function System of The Feeding Device

Now the motion equations of the feeding device will be established. Dynamic and kinematic scheme of the device is shown in Fig. 5 where m_1, m_2, m_3, m_4, I_1, I_2, I_3, and I_4 are masses and moments of inertia with respect to axis through gravity center of linkage bar 1, 2, 3, and 4 respectively. P is projectile gravity force and F is exterior force (controlled force rule in hydraulic motor hoisting the projectile into the required elevation angle).

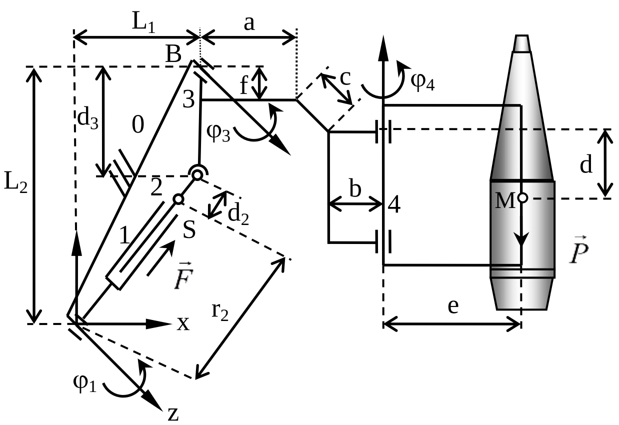

Fig. 5 Dynamic scheme of the feeding device with 2 DOF

Point S is gravity center of hydraulic piston 2 in the hydraulic motor.

The generalized coordinates are

$$\mathbf{q}=\begin{bmatrix}\varphi_1 & r_2 & \varphi_3 & \varphi_4\end{bmatrix}^T=\begin{bmatrix}q_1 & q_2 & q_3 & q_4\end{bmatrix}^T. \tag{29}$$

The coordinates of linkage bars gravity centers are set as follows:

$$\begin{aligned}
&x_1=l_1\cos q_1;\ y_1=l_1\sin q_1;\ z_1=0;\ \varphi_1=q_1\\
&x_2=q_2\cos q_1;\ y_2=q_2\sin q_1;\ z_2=0;\ r_2=q_2\\
&x_3=L_1+l_3\cos q_3;\ y_3=L_2+l_3\sin q_3;\ z_3=0;\ \varphi_3=q_3\\
&x_4=L_1+(a+b+e\cos q_4)\cos q_3;\ \ a+b=a_1\\
&y_4=L_2-f-d+(a+b+e\cos q_4)\sin q_3;\\
&z_4=c-e\sin q_4;\ \varphi_4=q_4.
\end{aligned} \tag{30}$$

The translational Jacobian matrices used in (17) are expressed as:

$$\mathbf{J}_{T1}=\frac{\partial\mathbf{r}_1}{\partial\mathbf{q}}=\begin{bmatrix}-l_1\sin q_1 & 0 & 0 & 0\\ l_1\cos q_1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, \tag{31}$$

$$\mathbf{J}_{T2}=\frac{\partial\mathbf{r}_2}{\partial\mathbf{q}}=\begin{bmatrix}-q_2\sin q_1 & \cos q_1 & 0 & 0\\ q_2\cos q_1 & \sin q_1 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, \tag{32}$$

$$\mathbf{J}_{T3}=\frac{\partial\mathbf{r}_3}{\partial\mathbf{q}}=\begin{bmatrix}0 & 0 & -l_3\sin q_3 & 0\\ 0 & 0 & l_3\cos q_3 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, \tag{33}$$

$$\mathbf{J}_{T4}=\frac{\partial\mathbf{r}_4}{\partial\mathbf{q}}=\begin{bmatrix}0 & 0 & -(a_1+e\cos q_4)\sin q_3 & (a_1-e\sin q_4)\cos q_3\\ 0 & 0 & (a_1+e\cos q_4)\cos q_3 & (a_1-e\sin q_4)\sin q_3\\ 0 & 0 & 0 & -e\cos q_4\end{bmatrix}. \tag{34}$$

The rotational Jacobian matrices used in (17) as well are written as follows:

$$\mathbf{w}_1=\begin{bmatrix}0\\0\\\dot{q}_1\end{bmatrix}\Rightarrow\mathbf{J}_{R1}=\frac{\partial\mathbf{w}_1}{\partial\dot{\mathbf{q}}}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\1&0&0&0\end{bmatrix}, \tag{35}$$

$$\mathbf{w}_2=\begin{bmatrix}0\\0\\\dot{q}_1\end{bmatrix}\Rightarrow\mathbf{J}_{R2}=\frac{\partial\mathbf{w}_2}{\partial\dot{\mathbf{q}}}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\1&0&0&0\end{bmatrix}, \tag{36}$$

$$\mathbf{w}_3=\begin{bmatrix}0\\0\\\dot{q}_3\end{bmatrix}\Rightarrow\mathbf{J}_{R3}=\frac{\partial\mathbf{w}_3}{\partial\dot{\mathbf{q}}}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\end{bmatrix}, \tag{37}$$

$$\mathbf{w}_4=\begin{bmatrix}0\\\dot{q}_4\\0\end{bmatrix}\Rightarrow\mathbf{J}_{R4}=\frac{\partial\mathbf{w}_4}{\partial\dot{\mathbf{q}}}=\begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}. \tag{38}$$

The inertia matrix **M** is:

$$\mathbf{M(q)}=\sum_{i=1}^{4}\left(m_i\mathbf{J}_{Ti}^T\mathbf{J}_{Ti}+\mathbf{J}_{Ri}^T I_i\mathbf{J}_{Ri}\right) \tag{39}$$

and after arrangements the inertia matrix is given by the next formula:

$$\mathbf{M}(\mathbf{q})=\begin{bmatrix} m_1l_1^2+m_2q_2^2 \\ +I_1+I_2 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3l_3^2+m_4(a_1+e\cos q_4)^2 \\ & & +I_3 & 0 \\ 0 & 0 & 0 & m_4e^2\cos^2 q_4 \\ & & & +I_4 \end{bmatrix} \quad (40)$$

According to (27) it is calculated the derivation of function matrix $\mathbf{M}(\mathbf{q})$ with respect to $\mathbf{q}$ as follows:

$$\frac{\partial \mathbf{M}(\mathbf{q})}{\partial \mathbf{q}}=\left[a_{ij}\right], i=1,...,4;\ j=1,...,16\ ,$$

where:

$$a_{11}=0;\ a_{12}=2m_2q_2;\ a_{1j}=0;\ j=3,...,16$$

$$a_{2j}=0;\ j=1,...,16$$

$$a_{3j}=0;\ j=1,...,11;\ a_{3;12}=-2m_4(a_1+e\cos q_4)\sin q_4$$

$$a_{3j}=0;\ j=13,...,16$$

$$a_{4j}=0;\ j=1,...,15;\ a_{4;16}=-2m_4e^2\cos q_4\sin q_4\ .$$

Using Kronecker product, according to (27), we calculate the Coriolis/Centripetal matrix $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$, see [17]:

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})=\begin{bmatrix} 2m_2q_2\dot{q}_2 & 0 & 0 & 0 \\ -m_2q_2\dot{q}_1 & 0 & 0 & 0 \\ 0 & 0 & -2m_4(a_1+e\cos q_4)\sin q_4\dot{q}_4 & 0 \\ 0 & 0 & m_4(a_1+e\cos q_4)\sin q_4\dot{q}_3 & -m_4e^2\cos q_4\sin q_4\dot{q}_4 \end{bmatrix} \quad (41)$$

The potential energy of the feeding device is given:

$$\Pi=\frac{1}{2}m_1gl_1\sin q_1+\frac{1}{2}m_2gq_2\sin q_1+\frac{1}{2}m_3g(L_2-l_3\sin q_3)+$$
$$+\frac{1}{2}m_4g[L_2-f-d+(a+b+e\cos q_4)\sin q_3]. \quad (42)$$

By derivation of the potential energy is determined member $\mathbf{g}(\mathbf{q})$:

$$\mathbf{g}(\mathbf{q})=\left(\frac{\partial \Pi}{\partial \mathbf{q}}\right)^T=\frac{1}{2}\begin{bmatrix} \left(m_1gl_1+m_2gq_2\right)\cos q_1 \\ m_2g\sin q_1 \\ -[m_3gl_3-m_4g(a_1+e\cos q_4)]\cos q_3 \\ -m_4ge\sin q_3\sin q_4 \end{bmatrix}. \quad (43)$$

The constraint equations of linkage bars are written as:

$$f_1=(r_2+d_2)\cos q_1-d_3\cos q_3-L_1=0$$
$$f_2=(r_2+d_2)\sin q_1+d_3\sin q_3-L_2=0$$
$$\mathbf{f}=\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}=\begin{bmatrix} (q_2+d_2)\cos q_1-d_3\cos q_3-L_1 \\ (q_2+d_2)\sin q_1+d_3\sin q_3-L_2 \end{bmatrix}$$

$$\Rightarrow \mathbf{J}_{\mathrm{f}}=\frac{\partial \mathbf{f}}{\partial \mathbf{q}}=\begin{bmatrix} -\left(q_2+d_2\right)\sin q_1 & \cos q_1 & d_3\sin q_3 & 0 \\ \left(q_2+d_2\right)\cos q_1 & \sin q_1 & d_3\cos q_3 & 0 \end{bmatrix}, \quad (44)$$

$\boldsymbol{\lambda}=\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$ is vector of Lagrange multipliers

$$\mathbf{f}_2=-\boldsymbol{\lambda}^T\frac{\partial \mathbf{f}}{\partial \mathbf{q}}=-\mathbf{J}_{\mathrm{f}}^T\boldsymbol{\lambda}=\begin{bmatrix} -\left(q_2+d_2\right)\lambda_1\sin q_1+\left(q_2+d_2\right)\lambda_2\cos q_1 \\ \lambda_1\cos q_1+\lambda_2\sin q_1 \\ d_3\lambda_1\sin q_3+d_3\lambda_2\cos q_3 \\ 0 \end{bmatrix}. \quad (45)$$

By substituting (40), (41), (43), and (44) into (28) we receive motion equation system of the device as follows:

$$\left(I_1+I_2+m_1l_1^2+m_2q_2^2\right)\ddot{q}_1+2m_2q_2\dot{q}_2\dot{q}_1+\left(m_1gl_1+m_2gq_2\right)\cos q_1$$
$$-\left(q_2+d_2\right)\lambda_1\sin q_1-\left(q_2+d_2\right)\lambda_2\cos q_1=\tau_1$$
$$m_2\ddot{q}_2-2m_2q_2\dot{q}_1\dot{q}_2+m_2g\sin q_1-\lambda_1\cos q_1-\lambda_2\sin q_1=\tau_2$$

$$\left[I_3+m_3l_3^2+m_4(a_1+e\cos q_4)^2\right]\ddot{q}_3-2m_4(a_1+e\cos q_4)\sin q_4\dot{q}_4\dot{q}_3$$
$$-[m_3gl_3-m_4g(a_1+e\cos q_4)]\cos q_3-d_3\lambda_1\sin q_3-d_3\lambda_2\cos q_3=\tau_3$$

$$\left(I_4+m_4e^2\cos^2 q_4\right)\ddot{q}_4+m_4(a_1+e\cos q_4)\sin q_4\dot{q}_3^2$$
$$-m_4e^2\cos q_4\sin q_4\dot{q}_4^2=\tau_4 \quad (46)$$

$$(r_2+d_2)\cos q_1-d_3\cos q_3-L_1=0$$
$$(r_2+d_2)\sin q_1+d_3\sin q_3-L_2=0 \quad (47)$$

The equation of motion of the feeding device (46) and two constraint equations (47) have been used for finding unknowns q_1, q_2, q_3, q_4, λ_1, λ_2.

Structural characteristics of the device such as moment of inertia with respect to the global coordinate system located at point B in Fig. 5, mass of the device components are shown in the Table I. The characteristics have been calculated using ANSYS Workbench software.

Using axial transformation formulas it is possible completely to calculate moment of inertia of each link with respect to its rotational axis. These moments of inertia are used as input data for solving equation system (46) and (47).

In (46) τ_1, τ_2, τ_3, τ_4 there are exterior forces (controlled forces and moments) with respect to generalized coordinate respectively, see [19], [21], [22], [23], and [25]. Exterior forces are determined from results of dynamic analysis of feeding devices using hydraulic motors or measurement results of pressure in hydraulic cylinder.

TABLE I
STRUCTURAL CHARACTERISTICS OF LINKS

Symbol	Quantity	Value and unit
m_1	mass of link 1	4.117 kg
I_{1xx}	x-axis inertial moment of link 1	3.504e-2 kg.m^2
I_{1yy}	y-axis inertial moment of link 1	2.391e-2 kg.m^2
I_{1zz}	z-axis inertial moment of link 1	3.511e-2 kg.m^2
m_2	mass of link 2	2.092 kg
I_{2xx}	x-axis inertial moment of link 2	1.054e-2 kg.m^2
I_{2yy}	y-axis inertial moment of link 2	4.160e-2 kg.m^2
I_{2zz}	z-axis inertial moment of link 2	1.058e-2 kg.m^2
m_3	mass of link 3	18.663 kg
I_{3xx}	x-axis inertial moment of link 3	0.238e-2 kg.m^2
I_{3yy}	y-axis inertial moment of link 3	5.036e-2 kg.m^2
I_{3zz}	z-axis inertial moment of link 3	0.278e-2 kg.m^2
m_4	mass of link 4	16.077 kg
I_{4xx}	x-axis inertial moment of link 4	0.262e-2 kg.m^2
I_{4yy}	y-axis inertial moment of link 4	0.284e-2 kg.m^2
I_{4zz}	z-axis inertial moment of link 4	0.531e-2 kg.m^2
m_5	mass of projectile	43.500 kg
I_{5xx}	x-axis inertial moment of projectile	1.312e-2 kg.m^2
I_{5yy}	y-axis inertial moment of projectile	0.143e-2 kg.m^2
I_{5zz}	z-axis inertial moment of projectile	1.312e-2 kg.m^2
X_1	gravity center coordinate of link 1	-0.134 m
Y_1	gravity center coordinate of link 1	-0.257 m
Z_1	gravity center coordinate of link 1	3.425e-2 m
X_2	gravity center coordinate of link 2	-6.999e-2 m
Y_2	gravity center coordinate of link 2	-0.181 m
Z_2	gravity center coordinate of link 2	3.425e-2 m
X_3	gravity center coordinate of link 3	7.620e-2 m
Y_3	gravity center coordinate of link 3	-0.140 m
Z_3	gravity center coordinate of link 3	3.115e-2 m
X_4	gravity center coordinate of link 4	0.423 m
Y_4	gravity center coordinate of link 4	-0.169 m
Z_4	gravity center coordinate of link 4	1.710e-2 m
X_5	gravity center coordinate of projectile	0.53 m
Y_5	gravity center coordinate of projectile	-0.129 m
Z_5	gravity center coordinate of projectile	8.425e-2 m

V. DYNAMIC AND KINEMATICS SIMULATION OF THE FEEDING DEVICE USING ANSYS

The geometry model of the feeding device established in ANSYS is shown in Fig. 6. The motion stage scheme of the feeding device is represented in Fig. 7.

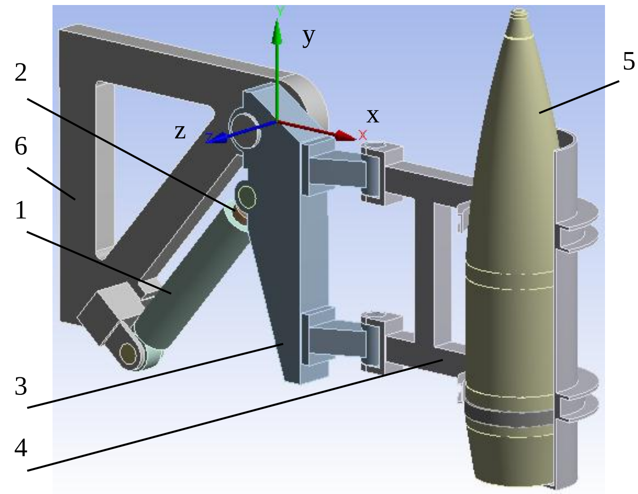

Fig. 6 Feeding device established in ANSYS
1- cylinder; 2- piston; 3- arm; 4- forearm; 5- projectile; 6- frame

The first stage: the forearm rotates in the horizontal plane to seize the projectile from the conveyer. The second stage: the forearm rotates in the reversal direction and takes the projectile into the vertical plane containing barrel axis. The third stage: the hydraulic cylinder-piston is operated and makes the projectile to rotate in the vertical plane to the required elevation angle.

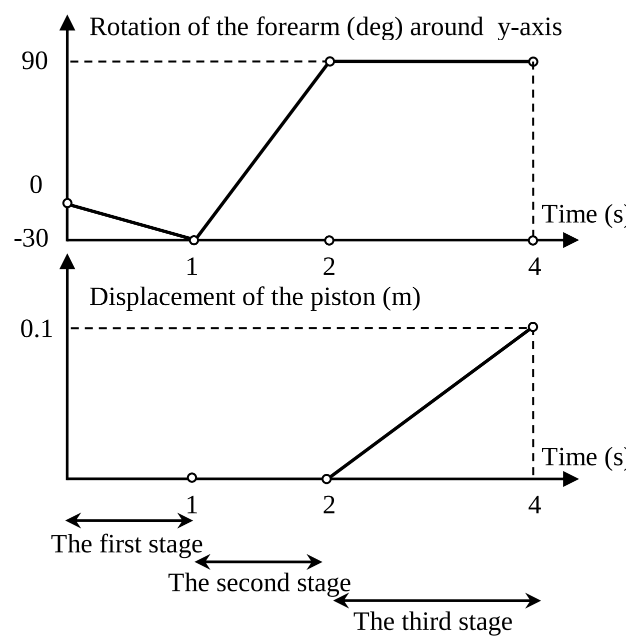

Fig. 7 Motion stage scheme of the feeding device

After simulation of the feeding device with input data shown in Fig. 7 the results are depicted in Fig. 8 – Fig. 15.

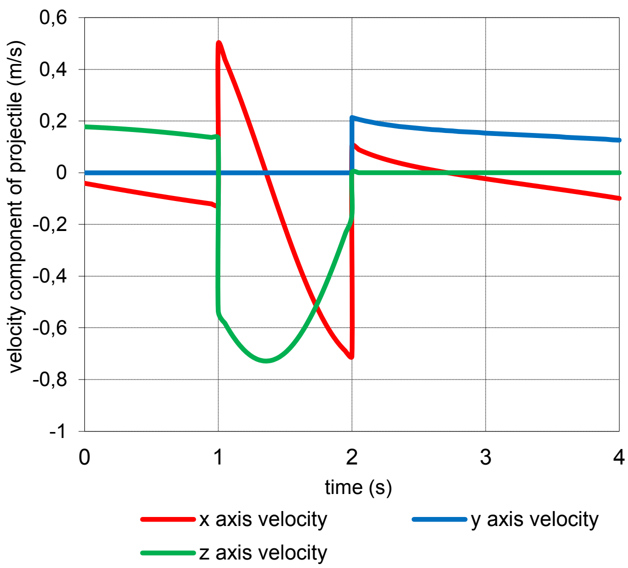

Fig. 8 Velocity of projectile

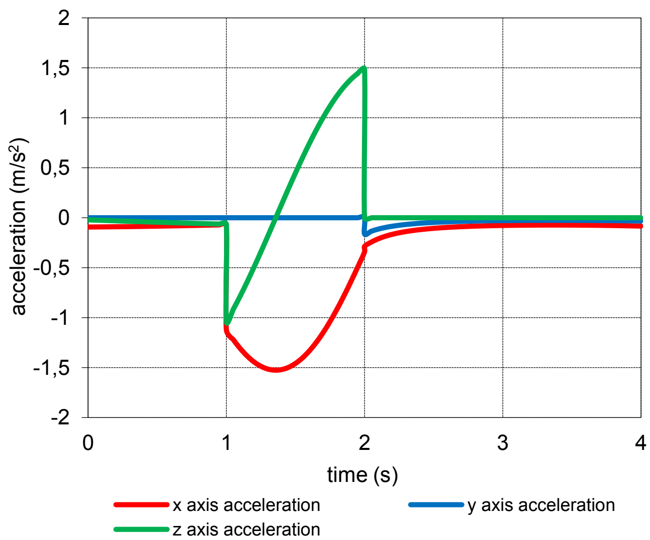

Fig. 9 Acceleration of the projectile

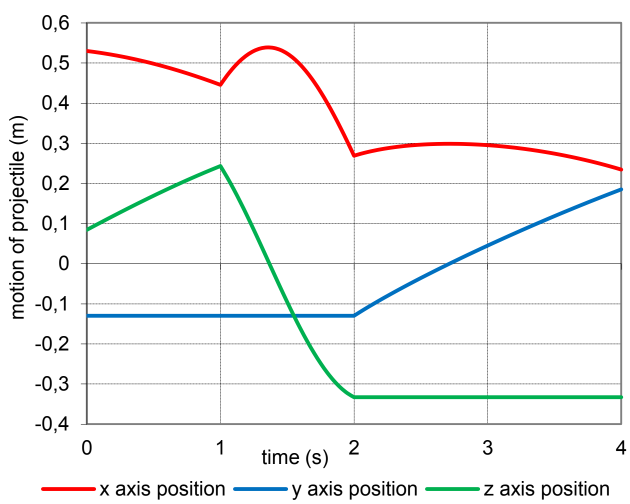

Fig. 10 Displacement of projectile during feeding process

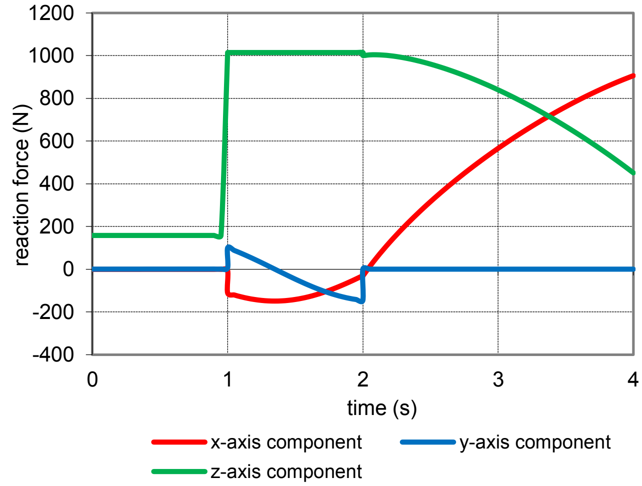

Fig. 11 Reaction force of forearm revolute joint versus time

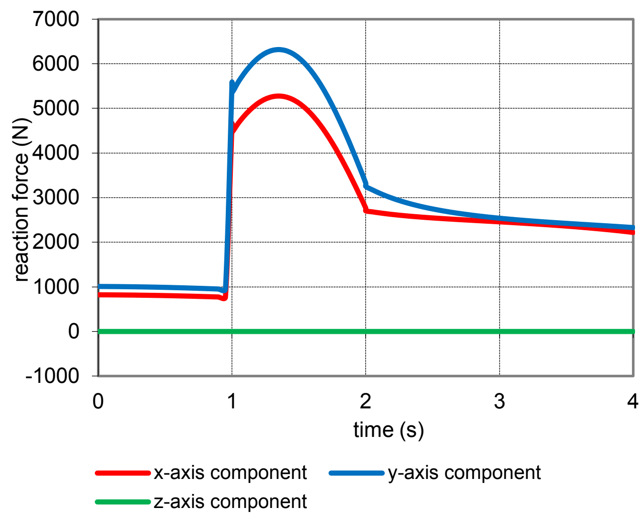

Fig. 12 Reaction force of revolute joint between cylinder and frame

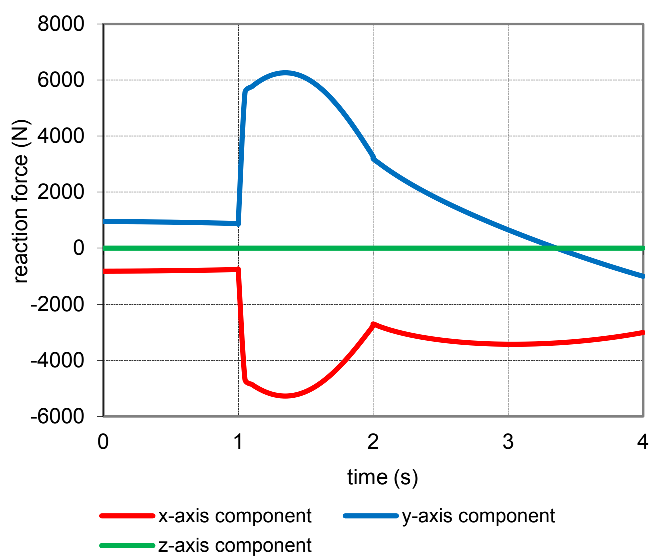

Fig. 13 Reaction force of revolute joint between piston and arm

Motion of projectile in Fig. 10 corresponds with input data shown in Fig. 7. For example y-axis motion, from zero to 2 second, y-coordinate is const, because projectile moves in horizontal plane. Afterwards the projectile moves in the vertical plane, so the y-axis coordinate increases to the maximal value where projectile is pushed into the cartridge chamber by the ramming device.

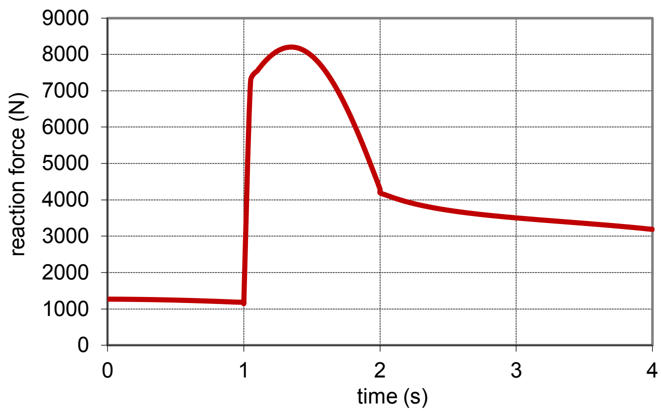

Fig. 14 Reaction force onto the axis of piston

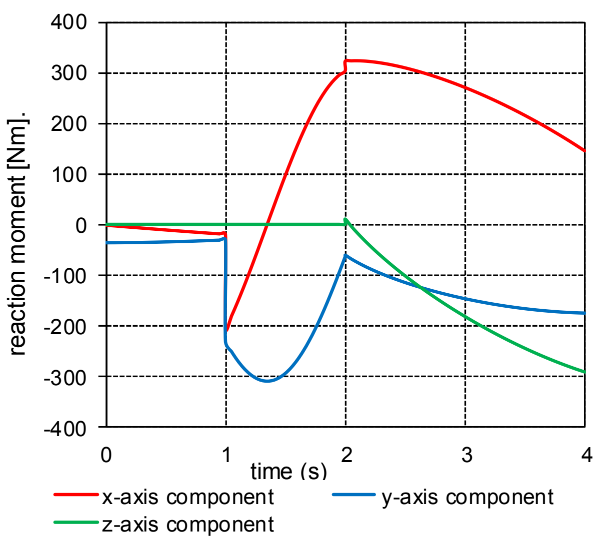

Fig. 15 Reaction moment of the revolute joint between forearm and arm

It is possible to use reaction force onto axis of piston presented in Fig. 14 and reaction moment of revolute joint between forearm and arm in Fig. 15 as controlled force for the dynamic problem. In this case instead of using input data as motion rules in Fig. 7 the input data are used as the force rule affecting onto the piston, force direction coincides with piston axis and moment rule that affects onto the revolute joint between forearm and arm.

It is possible also to use the linear velocity rule of the piston and rotary velocity rule of the forearm as input data for the simulation problem.

In figures of the reaction force and moment period of time from 0 until 1 second there is the feeding device without the projectile. So values of the reaction force or moment are quite small. In this stage, the forearm of the device performs rotary motion to seize projectile or propellant charge from the conveyer. All stages are identical for both projectile and case with propellant charge movements.

VI. Conclusion

As the results of the calculations, we have obtained the courses of the main kinematical parameters, working space of the feeding mechanism, and the reaction forces in the feeding devices. The article broadens out possibilities of feeding devices designing using mechanics which has been applied in the robotics and manipulators theory, see [6], [7], [8], and mainly [17].

As it is known, it is quite difficult to solve the dynamic problem of the specific industrial robot or manipulator as the weapon feeding device is. In many cases, although we have established motion derivative equation system of the robot, we also have to determine not only structural characteristics of the robot, such as linkage bar moments of inertia, gravity center coordinates, mass, but also exterior force rules (controlled rules), that ensures the robot operates as it has been designed. As a frame of this article the Rigid Dynamic component of the simulation software ANSYS has been used to solve the dynamic and kinematic problem of the feeding device. The simulation component operates on base of ANSYS-ADAMS interface, to provide user simulation tool for simulating multi-body system effectively.

The procedures used in this article will be applied in the Czech research institutes and in the University of Defence in Brno as additional teaching material for students of weapons and ammunition branch. Despite of this system is not up to date, the theory can be used during retrofitting of the system new propellant charges or fuses which change dimensions and weight and can caused malfunctions during loading due to greater length mainly loading automatically.

The continuation of there presented results will be connected with the research of the weapon barrels as it begins in [11], [12], [13], and [14].

This method enables to carry out analysis of vehicle vibrations on the loading process causing by the fire or during moving on the battlefield, see [1].

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Jiri Balla born in Poprad (Czechoslovakia), 6th June 1954. MSc degree in mechanical engineering at Military academy in Brno 1978. PhD degree in field weapons and protection against them at Military academy in Brno 1986. Assoc Prof of Military academy in Brno 1998 in field military technology, weapons and ammunition. Professor of Defense University in Brno 2006 in same field as Assoc Prof. Current the author's major field of study is dynamics of weapon barrel systems. He worked in military units as ordnance officer. After PhD studies he was a teacher as lecturer and associate professor.

He was visiting fellow at Royal Military College and Science (RMCS) in Shrivenham (UK) 1996, 1997, 1998. Currently he is a professor at University of Defense in Brno at Weapons and ammunition department.

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Prof. Balla is member of Czech Association of Mechanical Engineers (CzAME).

Van Yen Duong born in Vinh Phuc (Vietnam), 22nd February 1974. Eng. in field of weapon and ammunition at Military Technical Academy in Hanoi 1996. MSc degree in mechanical engineering at Military Technical Academy in Hanoi 2004. After university graduation, he is an officer in Vietnamese Army and has worked as a researcher at Weapon Institute in Hanoi for ten years in field of designing weapon and ammunition system. Currently, he is studying PhD level in field of weapons and protection against them in University of Defense in Brno, Czech Republic.

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