A comparative analysis between the vehicles' passive and active suspensions

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***Abstract*—**This paper presents a comparative analysis between the passive and active suspension systems of the motor vehicles. The study is performed for a half-car model, which corresponds to the guiding - suspension system of a rear axle. The active suspension system is obtained by placing a force actuator in parallel to passive suspension, the goal being to minimize the effect of the road disturbances. The passive and active suspensions are analyzed in the passing over bumps dynamic regime. The response of the active suspension is compared with the passive suspension, important improvements in the dynamic behavior (in terms of stability and comfort) being observed for the active suspension.

***Keywords*—**control system, dynamics, suspension, vehicles.

I. Introduction

THE passive suspensions have inherent limitations as a consequence of the choice of elastic and damping characteristics to ensure an acceptable behavior for the entire working frequency range. The need to obtain a compromise between the conflicting requirements among different vibrations modes of the vehicle justify the research of the active suspension systems, the intelligence being determined by a controller that takes data from the vehicle dynamics. The response of the passive suspension (fig. 1,a) is only affected by the external excitations and the system parameters that have a direct action on the suspension. Instead, the intelligent suspension (fig. 1,b) is affected by indirect parameters, such as acceleration of the roll, pitch or vertical oscillations. The way to implement intelligence in a suspension system is to use variable damping – semi-active suspension (fig. 2,a), or to create a counter-force system – active suspensions (fig. 2,b).

Active suspensions use separate actuators, which exert an independent force on the suspension for improving the dynamic behavior. When designing an active suspension, two important issues must be considered: the possible failure of the external energy source, and the transfer of a large quantity of mechanical energy in a structure that has the potential to destabilize the controlled system. However, the active suspension systems significantly improve car comfort, handling performance and driving safety, realizing an improved compromise among different vibrations modes of the vehicle (bounce, roll, pitch) [1]–[8].

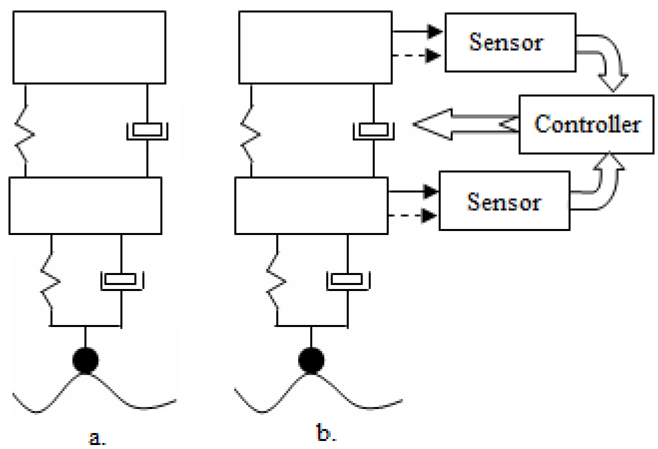

Fig. 1 Passive (a) and intelligent (b) suspensions

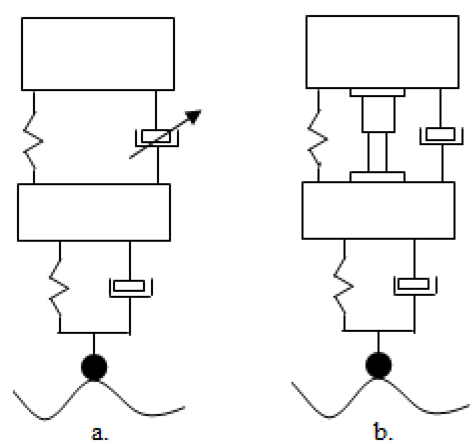

Fig. 2 Semi-active (a) and active (b) suspensions

On the other hand, the guiding - suspension system of the motor vehicles is a constrained, multi-body spatial mechanical system, in which the bodies are connected through geometric constraints, compliant joints (bushings), and force elements (springs, dampers, bumpers, tires). The complexity of the system makes it very difficult to handle the error in the case of simultaneous achievement of the entire model (full-vehicle

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model), and for this reason, in preliminary stages, partial models (quarter-car and half-car models) are developed - tested [9]–[14]. Usually, the quarter-car model is used to evaluate the vertical motion of the car body, the body equilibrium being ensured with a translational joint between the car body and ground, in the median plane of the vehicle. The half-car model is frequently used for studying the vertical motion as well as the roll and pitch oscillations.

Under these circumstances, the design and simulation of an active suspension system is approached in this paper. The analysis is performed for a half-car model, which corresponds to the guiding - suspension system of the rear axle. The active suspension is obtained by placing a force actuator in parallel to passive suspension, the objective being to minimize the effect of the road disturbances (which are considered as perturbations for the control system). The study is approached in the mechatronic concept, by integrating the multi-body model of the mechanical device and the control system model at the virtual prototype level [15], [16]. The virtual prototype of the active suspension system was developed using commercial software solutions: MBS (Multi-Body Systems) ADAMS - for modeling the mechanical device, and DFC (Design for Control) EASY5 - for the control system design.

II. Modeling The Suspension System

For this research, we have considered the half-car model which corresponds to the guiding - suspension system of a rear axle. In the lack of the front suspension, modeling a fictive joint between chassis and ground ensures the vehicle equilibrium. In order to evaluate the main motions of the car body, the equilibrium is made with a spherical joint on ground in the median - longitudinal plane (fig. 3).

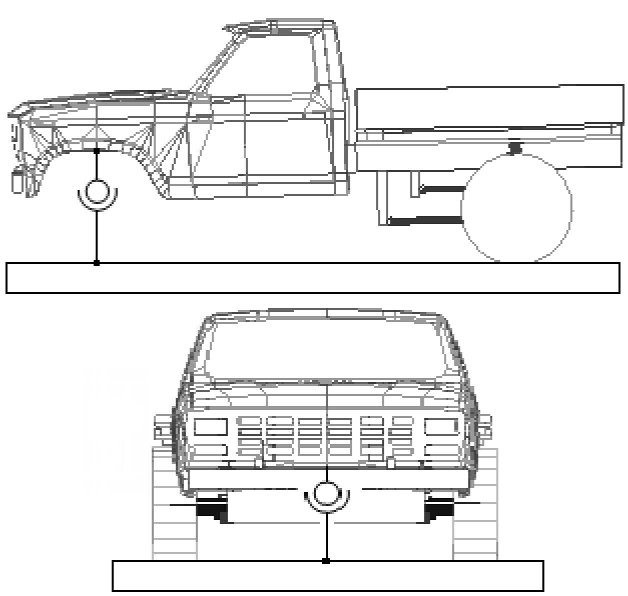


Fig. 3 The half-car model with spherical joint

The location of the spherical joint was obtained on the basis of double conjugate points' theory [17]. The property of these points is that forces applied at one of them produce no motion at the other. Each end of the car body makes an oscillation about its own conjugate point, so that the spherical joint for the rear axle model is placed in the front conjugate point. In this way, the half-car model takes into consideration the entire mass of the car body, and this is useful for the accuracy of the results.

The guiding mechanism in study is a dependent suspension model, the wheels being mounted at either end of a rigid beam so the movement of one wheel is transmitted to the opposite wheel causing them to steer and camber together; revolute joints connect the axle spindle to the rims. In relative motion to the car body, the rear axle is guided by a 4S-mechanism [18], with four longitudinal guiding bars, which are symmetrically disposed in two pairs (upper arms and lower arms) relative to the longitudinal axis of the vehicle (fig. 4). This model has been developed in the multi-body system concept [19]–[23], using the MBS software package ADAMS of MSC software.

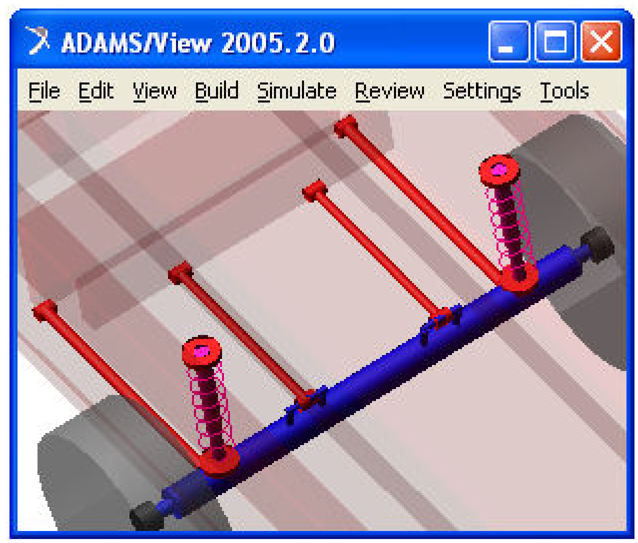


Fig. 4 The MBS model of the passive suspension system (4S-guiding mechanism)

The connections of the guiding bars to the adjacent parts (car body and axle) are made through compliant joints (bushings). The spring & damper groups are modeled as double active (tension and compression) elements of translational nature, being concentrically disposed between car body and axle. Beside the damping properties, the mass and inertia effects of the shock absorbers are also taken into consideration, and for this reason the lower and upper struts of the dampers have been modeled as rigid parts. In the virtual model, the damper components are connected by cylindrical joints, and the connections to the adjacent parts (car body and axle) are made through spherical joints.

The suspension system of the rear axle also contains bumpers and rebound elements. These are non-stationary elastic elements which limit the suspension displacement (compression - expansion), being disposed inside the dampers (the bumpers and the rebounds limit the relative displacement between the upper and lower struts of the dampers). These elements have been modeled as translational springs with unilateral rigidity, i.e. which are active only when elastic elements are in tension or in compression.

By placing the force actuators in parallel to the passive suspension, the active suspension system is obtained. The simplified model of the half-car active suspension is shown in fig. 5, with the following notations: m_2 - the sprung mass (car body); m_l - the unsprung mass (rear axle assembly - axle, wheels, guiding mechanism); k_2, c_2 - the stiffness and damping coefficients of the passive springs and shock absorbers; k, c - the stiffness and damping properties of the tires; $z_{l/r}$ - the road disturbances for the left/right wheels; $u_{l/r}$ - the counter-forces generated by the left/right actuators. For this paper, we have considered the following values of the parameters: m_l=120 kg, m_2=940 kg, k=255000 N/m, c=2500 Ns/m, k_2=45000 N/m, c_2=3500 Ns/m; these values correspond to the suspension system of a domestic motor vehicle.

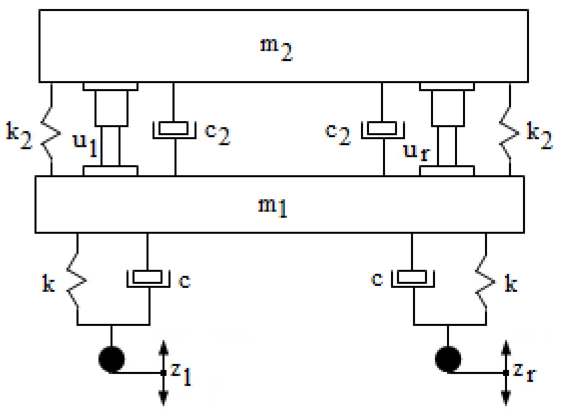


Fig. 5 The simplified half-car model of the active suspension

The control strategy is developed considering the road disturbances $z_{l/r}$ as perturbations to be eliminated by the control system, which generates the force signals $u_{l/r}$. This is a MIMO (Multi-Input Multi-Output) system (fig. 6), having as inputs the road disturbances and the force control signals, while the outputs are the wheels travels and the roll, pitch & yaw angles of the car body (considering the half-car model shown in fig. 3). Considering the superposition principle, the outputs are the combined effect of the input signals. The active suspension uses sensors to measure the vehicle dynamics, the output signals from the sensors being transmitted to the controller, which communicates with the force actuator.

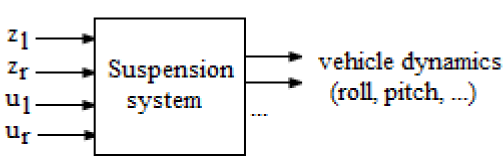


Fig. 6 The basic scheme of the MIMO system

The control system of the active suspension for the half-car model is developed by extending the control strategy for the quarter-car model that was described in [24]. The road disturbance (z) and the force control signal (u) are the inputs in the quarter-car model, while the output is modeled as the relative displacement (z_2-z_1) between the sprung (car body) and unsprung (wheel assembly) masses (fig. 7).

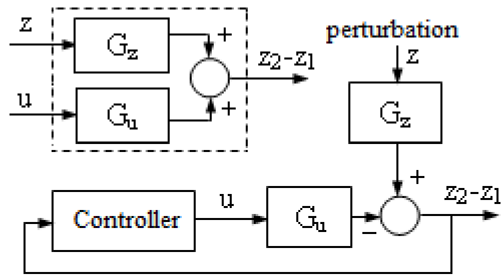


Fig. 7 The control system scheme for the quarter-car model [24]

In the case of the half-car model, the minimization of the effects due to road disturbances (to stabilize roll, pitch, and yaw response) is obtained by correlating the action of the left and right actuators. In these terms, a more complex control system is required, the strategy being based on a stability augmentation system (SAS) [25], [26], which is similar with the aircraft control [27]. The control system contains two control loops, in a cascade configuration with outer and inner loops. The inner loop is used to minimize the effect of the road disturbances (ride control), while the outer control loop is used to stabilize the response of the car body (attitude control). In addition, an input-decoupling transformation (IDT) is implemented for blending the control actions.

In this paper, because the yaw oscillation is very small, the study is focused on the stabilization of the roll and pitch oscillations. The inputs in the outer loop controller are the roll and pitch rates, while the outputs are the equivalent forces/torques to stabilize body roll (τ_R) and pitch (τ_P). The stabilization torques are distributed into active suspension using the IDT scheme shown in fig. 8. The outputs of the IDT block are then combined with the outputs of the inner loop controller to produce the control counter-forces in actuators (u_l and u_r).

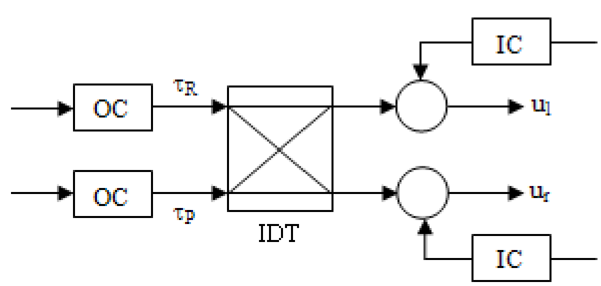


Fig. 8 The input-decoupling transformation (OC/IC – outer/inner loop controller)

The block diagram of the control system for the active suspension has been developed using the DFC (Design for

Control) software solution EASY5. For connecting the MBS mechanical model (see figure 3) with the actuating & control system, the input and output plants of the mechatronic system have been modeled in ADAMS/Controls, in accordance with the schemes shown in figures 6-8. The control forces in the left & right actuators (u_l and u_r) represent the input parameters in the mechanical model of the suspension system, while the outputs transmitted to the outer and inner controllers are the roll & pitch angles and angular velocities, as well as the relative motion between the sprung (car body) and unsprung (wheels/axle assembly) masses. The road disturbances (z_l and z_r) are directly applied in the MBS model of the mechanical device.

For the input state variables, the run-time functions are 0.0 during each step of the simulation, because the control forces will get their values from the control system. The run-time functions for the input variables are defined using a specific function that returns the variable value, namely VARVAL(value). For the output variables, the run-time functions return the rotational and translational displacements of coordinate system markers attached to car body around/along the motion axes of another markers attached to ground and axle, respectively. The next step is facilitating the exporting of the ADAMS plant files for EASY5, using the ADAMS/Controls plug-in. The input and output information are saved in a specific file for EASY5 (*.inf); ADAMS/Controls also generates a command file (*.cmd) and a dataset file (*.adm) which are used during simulation.

For controlling the active suspension, we have designed PID-based regulators (fig. 9). Lead-Lag block models a continuous first order transfer function, using the time constant format. The numerator time constant (TC1) is used as a lag time constant to calculate an approximate derivative from the signal, while the denominator time constant (TC2) is used to help prevent an implicit loop. The time constant represents the time it takes the system's response to reach 1-1/e ≈ 63,2% of its final value, where "e" is the mathematical constant (e= 2,71828.....).

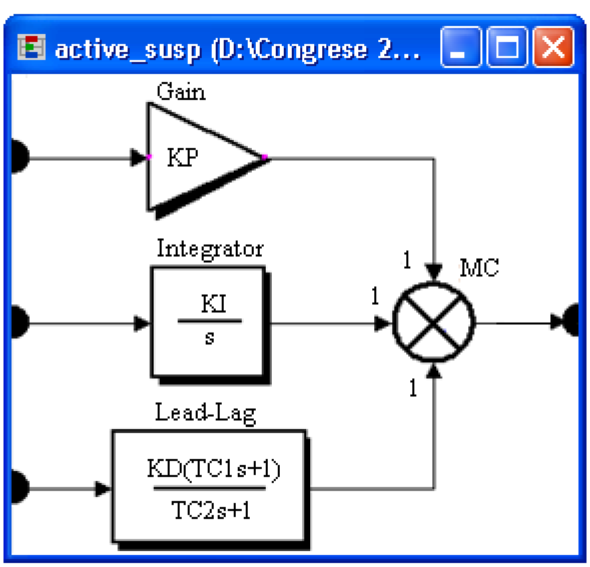


Fig. 9 The block model of the controller

III. Optimizing The Control System

The tuning of the controllers, in order to establish the optimal values of the gain factors (K_P - proportional gain, K_I - integral gain, K_D - derivative gain), was made by performing a parametric optimization with the Matrix Algebra Tool (MAT). This is an interactive tool which is specifically designed to be used in conjunction with EASY5. For beginning, the control system was exported as MAT EMX function. The design variables used in the optimization study are the gain factors of the controllers, while the design objective is to minimize the indicial response of the suspension.

The optimization is made in two stages: performing parametric studies, and optimizing the controller. The parametric studies represent sets of simulations that help to adjust a parameter to measure its effect on the performance of the controller, by sweeping the variable through a range of values and then simulating the behavior of the various designs in order to understand the sensitivity of the overall system to these design variations. As result, the parametric studies allow to identify the main design variables.

The parametric studies have been successively performed for each design variable, in the variation field "1,…,10^6". The study is based on a function script (*.ezemf) that contains the instructions used to setup and run multiple simulations: defining the plot and simulation commands (the instructions "setup_plot" and "setup_sim"), defining the loop used to change the gain values ("for"), defining the design variables to be used in the simulation ("sprintf"), loading the gain value, setting the plot parameters and the simulation parameters ("simulate"), loading the EASY5 plot data into MAT ("load_plot"), and setting the plot data ("label data", "sprint", "plot", drawplot"). In these terms, for step input signal with the amplitude 0.1 m, the results of the parametric studies are shown in figures 10-12.

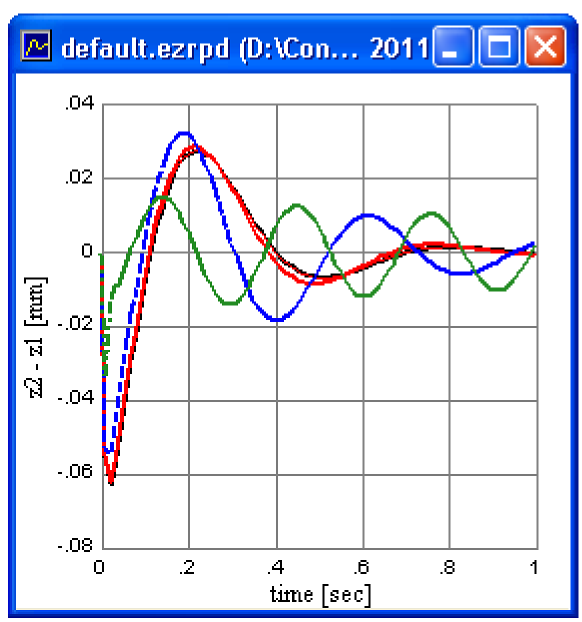


Fig. 10 The parametric study for the proportional gain (K_P)

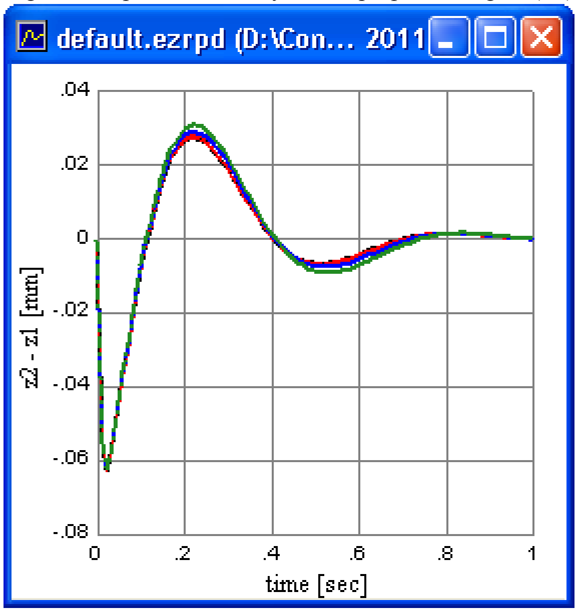


Fig. 11 The parametric study for the integral gain (K_I)

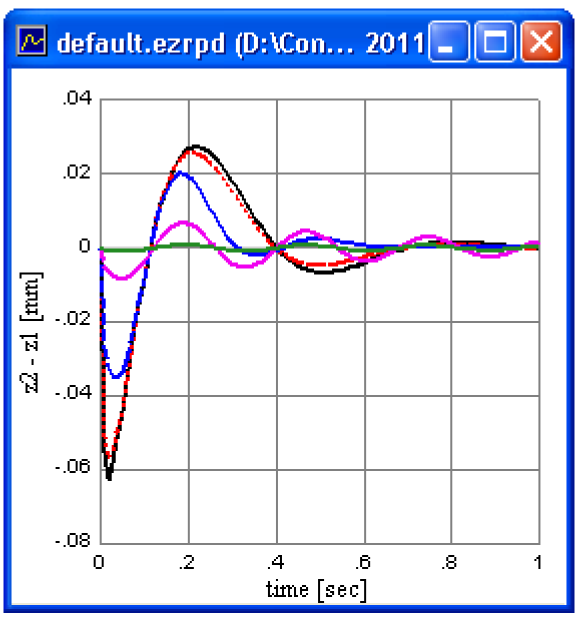


Fig. 12 The parametric study for the derivative gain (K_D)

From these results we can conclude that the main variables, with great influence on the design objective, are the proportional and derivative gains of the controller, while the integral gain is a secondary variable. Afterwards, the optimization study is performed considering the main design variables (K_P and K_D). For performing the optimization, the "minimize_v" function has been used. This function has the following syntax, [x,f] = minimize_v(funcname, x0, H0, tol, delx), where: funcname - name of function used to setup minimization, x0 - initial guess for minimizer, H0 - initial guess for Hessian, tol - relative tolerance for x, delx - relative step size for computing gradients by differencing, x - minimizer of the function, or last search position, f - value of the function at x.

The next step is to create a MAT function that will perform the optimization we require. This function will take the proportional and derivative gains of the controller model as inputs and return the root mean square of the indicial response of the active suspension. The script file that defines the minimize function contains MAT instructions for setting the values of the gains ("sprintf"), setting the initial time and conditions ("initial time", "initial conditions"), setting the integration time ("Tmax"), performing the integration ("calc_xic", "simulate"), getting the output value ("get_value"), and calculating the objective function to minimize ("mean", "sqrt").

The optimization is performed by calling the "minimize_v" function with the minimizer function as the first argument. MAT will repeatedly call the minimizer function as it performs the minimization procedure. The minimize function will set the proportional and derivative gains appropriately, perform a simulation, and retrieve the output value. The function returns the error in the simulations, defined as the difference between the simulation and desired value. The final values of the design variables result in a simulation that meets the design requirements - the settling time should be less than 2 seconds, while the maximum acceptable value for the overshoot is 5% [28], [29]: $K_P \cong 1.6e5$, $K_D \cong 2e5$; thus, a PD device is necessary, and sufficient, to control the suspension.

IV. Simulation Results

The half-car models in study, corresponding to the passive and active suspension systems, are analyzed in the passing over bumps regime, using a virtual testing stand (fig. 13). The simulator contains two linear actuators on which the rear wheels are anchored, the driving elements executing vertical motion relative to the fixed structure (rigidly attached to ground) for simulating the road profile.

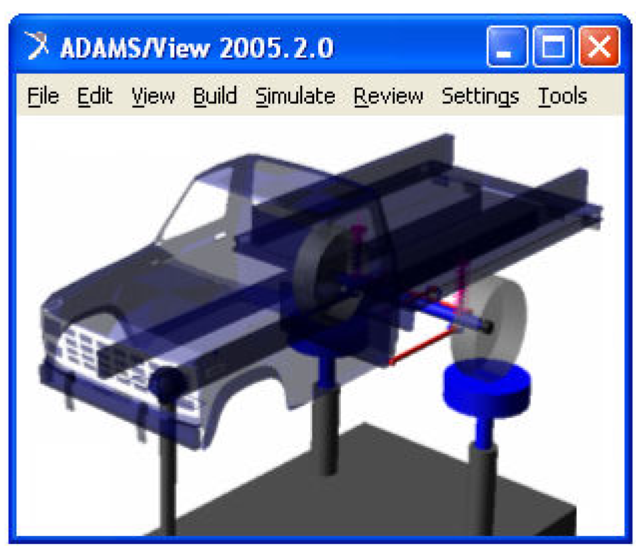


Fig. 13 The virtual testing stand for the half-car model

The connection between the wheels (tires) and the sustaining plates are made using contact forces, which are modeled as unilateral constraints (force that have zero values when no penetration exists between the specified geometries and positive values when penetration exists). The following data have been used to model the contact forces (i.e. the tires): the geometry of the wheels and tire patches, the tire stiffness coefficient, the force exponent, which is a real variable that specifies the exponent of the elastic characteristic (for a stiffening tire characteristic, e>1.0, and for a softening tire characteristic, 0<e<1.0), the damping properties of the contacting material, and the penetration depth, representing the boundary penetration at which the full damping is applied.

The inputs applied to the tire patches simulate the road profile. In order to evaluate the main motions of the car body (vertical, roll and pitch oscillations), the left wheel pass over a bump, while the right wheel "runs" on smooth surface (fig. 14). The modeling of the input signals for the left and right wheels was made using motion generators applied in the translational joints between actuators and ground. The motion law for the left wheel is defined by using a sum of STEP functions: STEP(time,0.0,0.0,0.25,50.0)+STEP(time,0.25,0.0, 0.5,-50.0). For the right wheel, the displacement function is zero during the whole simulation.

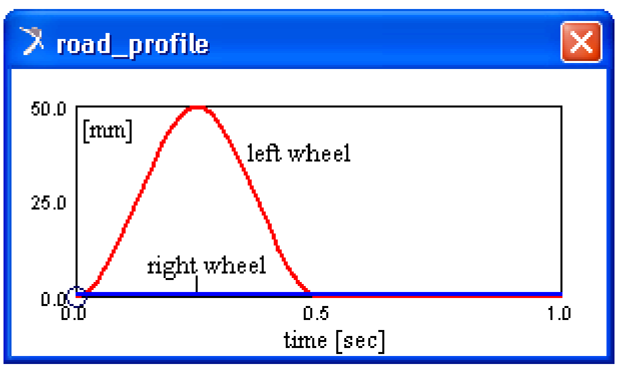


Fig. 14 The road profile for simulation

The response of the active suspension system is compared with the passive suspension, in the same testing conditions. In accordance with the diagrams shown in figures 15-18, important improvements in the dynamic behavior (in terms of stability and comfort) can be observed for the active suspension system. The improvement is also sustained by the numeric values shown in table 1, which represent the root mean square (RMS) of the interest parameters, and the graphic simulation frames from figure 19 (a - active suspension, b - passive suspension). These results demonstrate the viability of the design & control strategy for the active suspension.

Table 1

parameter \ RMS		passive suspension	active suspension
roll angle	[dgr]	0.9394	0.0578
pitch angle	[dgr]	0.2578	0.0273
roll velocity	[dgr/sec]	7.6409	0.4878

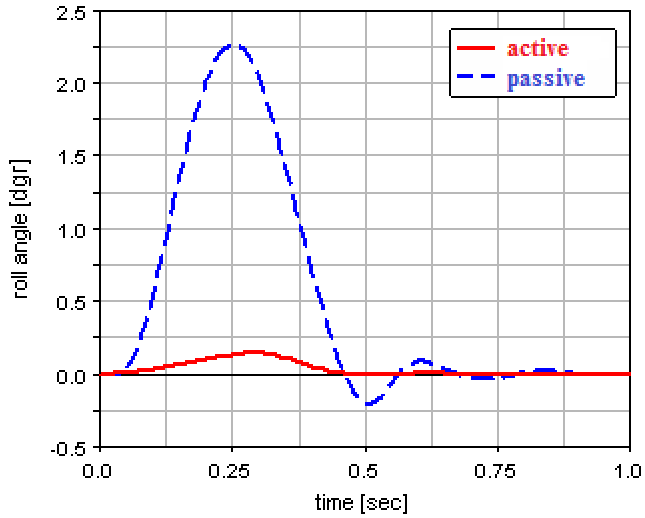


Fig. 15 The time-history variation of the roll angle

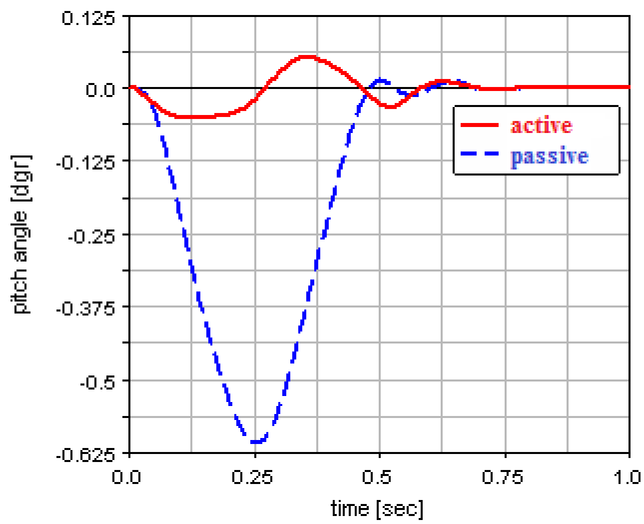


Fig. 16 The time-history variation of the pitch angle

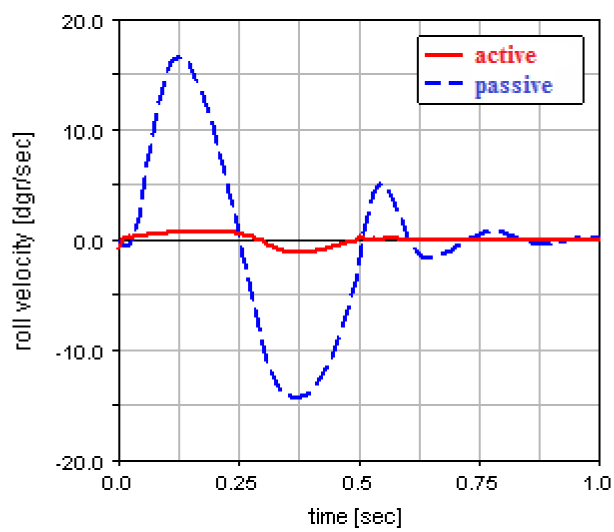


Fig. 17 The time-history variation of the roll velocity

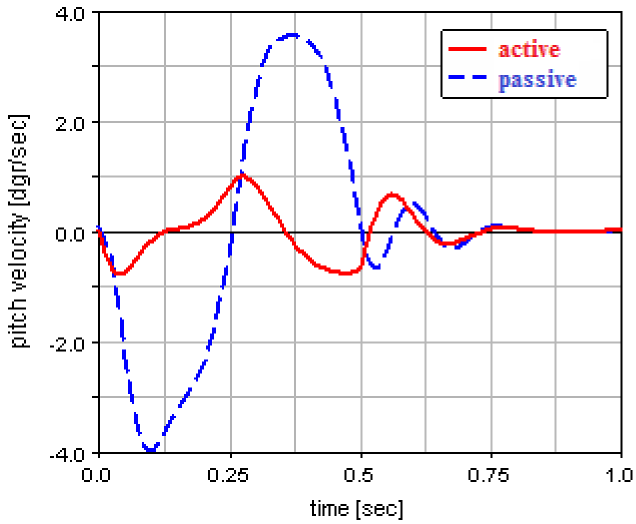
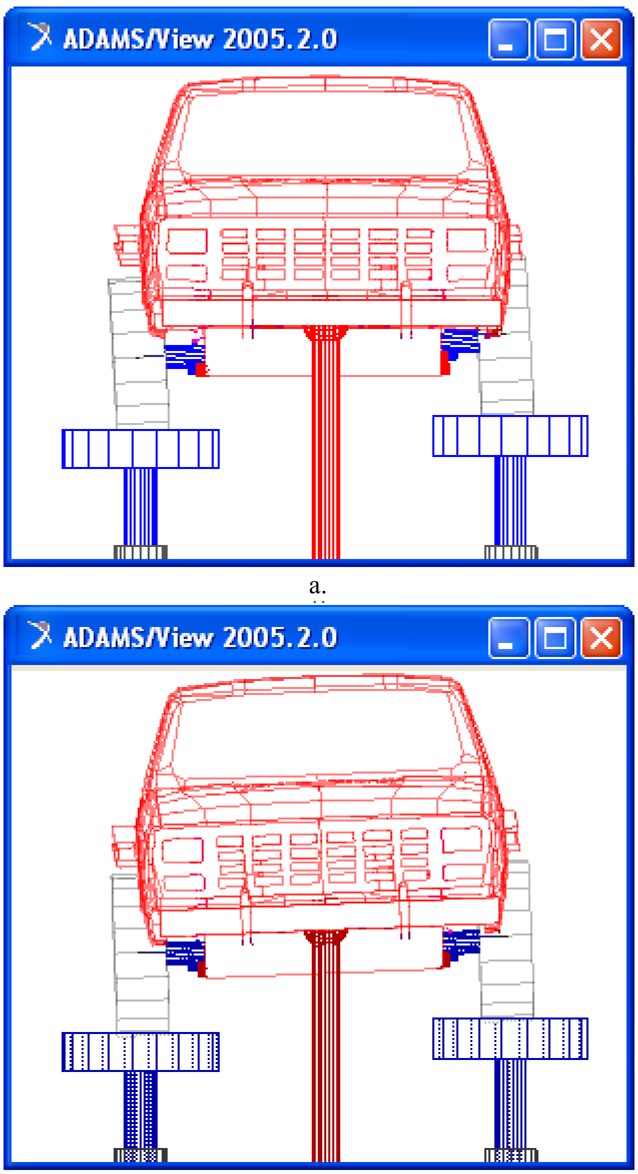


Fig. 18 The time-history variation of the pitch velocity



a.

b.

Fig. 19 Graphic simulation frames (a - active, b - passive)

The final issue is to verify the frequency response of the active suspension system, which is the measure of the system's output spectrum in response to the input signal. The frequency response is characterized by the magnitude of the system's response and the phase, versus frequency. We have obtained the frequency response by plotting the magnitude and phase measurements through the Bode plot, which is a graph of the transfer function of the system versus frequency, plotted with a logarithmic-frequency axis. In accordance with the plot shown in figure 20, the system acts as a filter, the maximum value of the resonance amplitude being -18 dB; consequently, the system filters out very well.

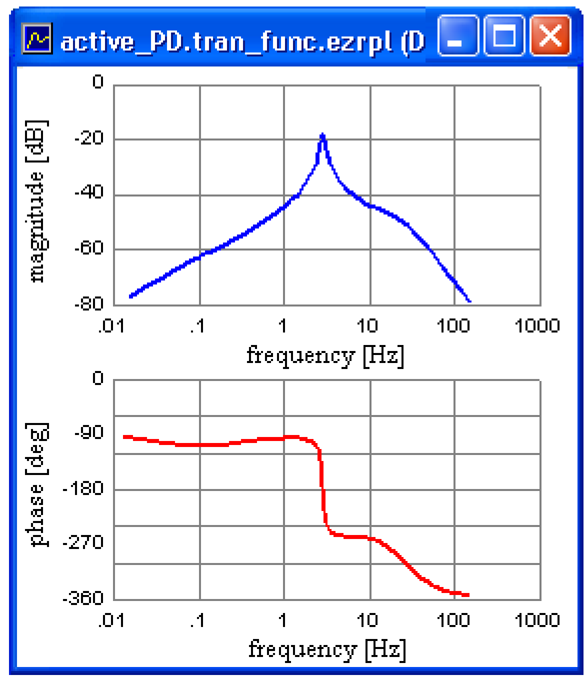


Fig. 20 The frequency response of the active suspension system

V. Final Conclusions

The application is a relevant example regarding the implementation of the virtual prototyping tools in the design process of the guiding - suspension systems. One of the most important advantages of this kind of simulation is the possibility to perform virtual measurements in any point or area, and for any parameter (motion, force). The future researches in the field will be focused on the research of more complex suspension models, such as the full-car model, and for other control strategies & controller types.

At the same time, the virtual prototype of the suspension system will be refined by modeling the mechanical structure with finite elements, for identifying the eigenshapes and eigenfrequencies. Integrating the finite element model in the multi-body system analysis, we can quickly build a parametric flexible body representation of a component, analyze the system, make changes to the flexible body and evaluate the effect of the changes. This will help us to take quick decisions on any design changes without going through expensive hardware (physical) prototype building and testing.

The active suspension system is in manufacturing stage, and it will be tested by using an experimental stand (fig. 21), creating a real perspective for the research in the field. The testing installation is based on hydraulic linear actuators (MTS HTC - Hydraulic Testing Components), including the control digital system Flex Test GT Controller and the applications Basic Test Ware & Multi-Purpose Test Ware. This will allow a relevant comparison between the virtual prototype analysis and the data achieved by measurements, in the validation process of the virtual prototype. The results of the experimental study will be presented in a future paper.

Fig. 21 The experimental testing installation

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